USE OF EXTREME CASES BY EXPERTS AND STUDENTS AS A LEARNING STRATEGY[1]

We present evidence from videotape transcripts for the productive use of extreme case reasoning by both scientifically trained experts and students. Extreme cases appear to be a powerful method for making inferences about physical systems but the origin of their power is difficult to explain. Spontaneous gestures and other imagery indicators provide evidence that they can involve the use of imagery in mental simulations as a possible source of power. The study identifies a reasoning process that can contribute to theory construction in scientists, and suggests that it may involve imagery and simulation in a central way. It attempts to describe an active nonformal reasoning process that students can do with some scaffolding, and outlines some of its strengths and limitations.

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INTRODUCTION

Objectives

We asked whether evidence for the generation and use of extreme cases by experts and by students could be documented from case studies, and if so, whether this might be associated with the use of dynamic mental imagery. We also asked whether extreme case reasoning could be scaffolded by a teacher during whole class discussion.

To investigate these questions, we analyzed videotapes of (1) scientifically trained experts and (2) secondary physics classroom discussions.

Theoretical Perspective

There has been little analysis of the roles that extreme case reasoning and imagery play in supporting model construction in the classroom. The ability to generate and evaluate mental models appears to be a crucial aspect of science and of student thinking (Darden, 1991; Gentner, 2002; Glynn and Duit, 1995; Hammer, 1995; Nersessian, 1995; Reiner and Gilbert, 2000; Vosniadou, in press), but Driver (1983) suggests that students often need to be helped to assimilate their prior experience into scientifically accepted models. Hegarty (1992) hypothesizes that a mechanism involved in subjects' evaluation of their mental models is the use

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of mental animation to run the models. Weld (1990) proposed that one mechanism for the effectiveness of an extreme case is to allow access to the second of two data points (pairs) for the values of two related variables, but Clement (2008) has offered a different hypothesis: that extreme cases can serve as a heuristic for enhancing imagistic simulations. To help speak to this question, one can use gesture data to provide at least a partial window onto subjects' mental imagery, as supported by the work of Goldin-Meadow (1999), Hostetter and Alibali (2004), and Hegarty, et al. (2005). For example, findings from Lozano and Tversky (2005) and Iverson and Goldin-Meadow (1997, 1998) suggest that the type and amount of gesture appear to be closely associated with the nature of the subject's internal representation. By analyzing gesture and other data, we believe there is much more that can be learned about the variety of reasoning processes students employ in the classroom to generate and evaluate their mental models.

PROCEDURE

Data Sources

Our definition for *extreme case reasoning* is a refinement of a definition originally developed during an earlier analysis of eleven videotapes of scientifically trained experts (professors or post-comprehensive exam graduate students in technical fields) who were asked to think aloud as they solved problems unfamiliar to them (Clement, 2008). Three expert episodes from these videotapes are presented as exemplars and are further analyzed here for evidence of mental imagery. Classroom case study data was drawn from two videotapes of classrooms using a curriculum that involved students in developing theories, models, and explanations for phenomena; several episodes from these studies are used here as exemplars.

Coding

We coded for the generation and use of extreme case reasoning and for the presence of gestures that appeared to depict an object or event. The expert study was coded by one of the authors and then used by both authors as an exemplar with which to further hone the criteria. Coding of classroom tapes was done jointly by consensus of the two authors, partly to help us refine the definitions of categories and to increase discriminatory power in the case of incomplete or unclear articulation by students. Thus, this study falls within a generative phase of the research in which we are attempting both to refine observational descriptors and to develop an initial theoretical framework within which to interpret them.

Coding for presence of Extreme Case Reasoning.

The criteria for coding for the presence of extreme case reasoning that has evolved from this process of refinement is:

> **An *Extreme Case has been generated*** when, in order to facilitate reasoning about a situation A (the target), a situation E (the extreme case) is suggested, in which some aspect of situation A has been maximized or minimized. This includes going almost to the end of a continuum for the aspect or well outside the normal range of the aspect.

> **An *Extreme Case has been run*** when, in order to facilitate reasoning about a situation A (the target), a subject gives evidence for reasoning about, making inferences from, or predicting from a situation E (the extreme case) in which some aspect of situation A has been maximized or minimized. The extreme case may initially have been suggested by a different subject.

Often when an extreme case is generated, it is also run. There are exceptions, however, as when a teacher proposes a case but leaves it to the students to consider.

Coding for presence of Mental Imagery.

The microanalysis of *depictive gestures,* which appear to depict an imaginary image "in the air" near the speaker, yielded one kind of indication that internal, or mental, imagery was being used.

Clement (2008) reviews a variety of studies of depictive gestures that suggest they are concurrent expressions of core meanings or reasoning strategies and not simply delayed translations of speech, justifying their role in providing evidence for the involvement of mental imagery. In particular, we are interested in evidence for the use of animated or *runnable* mental imagery, which we obtain from gestures that appear to depict an imaginary motion or force. Identifying these types of hand motions gives us a potential foothold on distinguishing between static and animated mental imagery. Examples and descriptions of additional imagery indicators will be provided below and a larger list of imagery indicators is provided in Clement (2008).

We jointly coded the videotapes for the presence of depictive gestures and the transcripts for the presence of kinesthetic imagery reports, another kind of imagery indicator discussed below.

Method of analysis.

We organized our data by *case*, *variation* of a case, and *episode*. A *case* is a concrete example of a system. A case introduced during a discussion about the causes of gravity, the US/Australia case, comprised the Earth, two people standing on it, and the gravitational forces between the Earth and the people. A *variation of a case* involved the same concrete example of a system but with some variable changed in a significant way (such as to create an extreme case) or with an additional variable highlighted. For instance, when a student introduced the rotation of the Earth into the discussion as a possible factor causing gravity in the US/Australia case, we counted this as a variation of that case. An *episode* involved a single student either generating or running a case or variation.

Individual episodes were examined to determine whether they met the criteria above for *extreme case reasoning*. We then noted which episodes were associated with depictive gestures or other imagery indicators. For the classroom transcripts, we also noted whether the student had generated the extreme case or was running a case that had been proposed by the teacher or by another student.

EXPERT CASE STUDY EXAMPLES

Examples discussed here come from transcripts of experts thinking aloud about the following "Spring Problem":

> A weight is hung on a spring (shown in Figure 1). The original spring is replaced with a spring made of the same kind of wire, with the same number of coils, but with coils that are twice as wide in diameter. Will the spring stretch from its natural length more, less, or the same amount under the same weight? (Assume the mass of the spring is negligible.) Why do you think so?

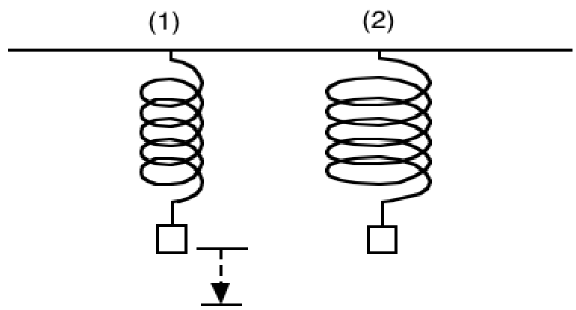

Figure 1. Spring Problem.

The simplest example of an extreme case comes from subject S3 comparing the narrow and wide springs.

Episode 1.

> "So the way to really *eke out my intuitions* would be to take the coiled spring in 1 down to an extremely tightly coiled spring. It's almost no distance from side to side of the spring. And obviously in that case it can't stretch very far…. *(I)f you...imagine* shrinking the coils to a very small diameter, the wire would be practically straight and you could barely stretch it at all. There'd be no 'give' to it."

The subject infers that narrower springs should stretch less. The subject's use of the term "eke out my intuitions" supports the interpretation of the extreme case as "enhancing the use of a physical intuition schema."

We use italicized type in the transcripts to identify segments providing some evidence for imagery use (both kinesthetic and visual). Here we hypothesize that this extreme case was used to enhance the use of a physical intuition in an imagistic simulation. (This is discussed further below.)

Episode 2.

In a second example, S7 generates the extreme case of a very wide spring:

> “If we had a case where the second one went– had huge diameters compared to the first, it would appear to sag a lot more. It just *feels like it would be a lot more spongy*.”

By changing the problem into an extreme case comparison, the subject is able to imagine a result, apparently kinesthetically. He then enhances the imagery still more—by imagining placing a very heavy weight on each spring.

> “*Imagine* putting a very heavy weight on it so it disturbs it a lot– that [very wide spring] *would seem a lot easier*– it would stretch more.”

The size of the equal weights in the problem is not specified and indeed, ones expectation is that changing this parameter should be completely irrelevant to the answer to the problem. So it is somewhat puzzling as to why the subject changes it. But one can hypothesize that it acts to enhance the imagery so that the effects in the system are larger and the imagined result becomes clearer. We call this imagery enhancement.

Episode 3a.

The last expert example is from a subject who hits upon the idea that the wire in the stretched spring is deforming by twisting rather than bending. He makes the analogy to twisting long and short rods:

> “If I have a longer rod [moves hands apart], and *I put a twist on* it [*moves hands as if twisting a rod*], it seems to me—again, physical intuition—that it will twist more…I'm [*raises hands in same position as before and holds them there continuously*] *imagining holding something that has a certain twistyness to it, a-and twisting it*…. ”

If this analogy is valid, it indicates that the wider spring would stretch more. An extreme case of twisting a very short rod is generated immediately after the statement above:

Episode 3b.

“Now I'm confirming [*moves right hand slowly toward left hand*] that, by using this method of limits. As [*moves right hand toward left hand until they almost touch at the word “closer*”] I bring my hand up closer and closer to the original place where I hold it, *I realize* very clearly that *it will get harder and harder to twist*. So that confirms my intuition, so I'm quite confident of that…. ” (See Figure 2.)

The reader may wish to try this thought experiment with images of thin coat hanger wire, bent to have 1 inch “handles” at each end of the wire.

A hypothesis that explains the process underlying this extreme case is the following. Given the above observations, it is plausible to interpret this process as “imagery enhancement” (or “simulation enhancement”)—that the role of this extreme case is to enhance the subject’s ability to run or compare imagistic simulations with high confidence.

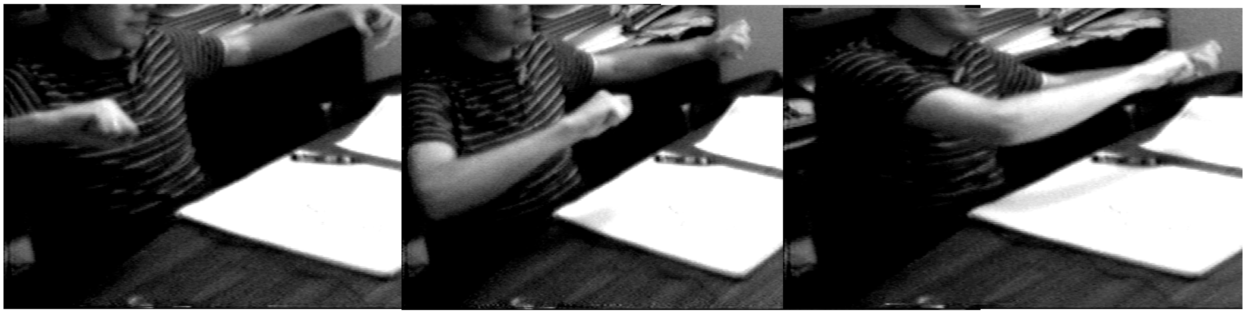

Figure 2 a, b, c. Extreme case comparison gestures for twisting a rod.

Imagistic Simulation Mechanism Underlying Extreme Cases

Evidence for imagery.

In this section, we unpack the above hypotheses more carefully and tie them to transcript data. Italicized type in the protocol episodes above identifies examples of several imagery-related observation categories, listed here in the order they occur in Episode 3a: personal action projections (spontaneously redescribing a system action in terms of a human action) consistent with the use of kinesthetic imagery, depictive gestures (gestures that depict objects, forces, locations, or movements of entities), and imagery reports. The latter occurs when a subject spontaneously uses terms like "imagining," "picturing" a situation, or "feeling what it's like to manipulate" a situation. In this last case, it is a dynamic imagery report (involving movement or forces). None of these observations are infallible indicators on their own but are used here as evidence for imagery, and this is reinforced when more than one appear. All indicators above except reports of static imagery are also evidence for *dynamic* imagery of the kind that could be used in a simulation. Such indicators appear alongside new predictions in the protocol segments, supporting the hypothesis that some type of internal imagistic simulation is occurring.

Schema-driven simulations.

One can also draw on the historical precedent of motor schema theory (Schmidt, 1982) in hypothesizing that analog perceptual motor knowledge structures that can control real actions over time (e.g. a schema for “twisting” objects) are involved here. The observations in Episode 3, for example, can be explained via what Clement (1994) has called a *schema-driven imagistic simulation*, as follows. (1) The subject has activated a somewhat general and permanent perceptual motor schema that can control the action of twisting real objects; the schema is capable of coordinating real actions and perceptions over time and does this partly by generating action command trajectories and perceptual expectations. This capability allows it to generate imagery of anticipated actions and perceptual expectations in the absence of real objects and actions, presumably by driving presymbolic activity top down in some of the upper layers of the perceptual and motor systems. (2) The schema assimilates images of two rods of different lengths that are more specific and temporary. (3) The schema “runs through an action” of twisting over time vicariously without touching real objects, generating an imagistic simulation of twisting each rod, and the subject compares the anticipated effort required for each. Kosslyn’s

(1980) theory of static imagery is followed in referring to this last step as image interrogation and inspection, here extended to images of events and actions.

Such a simulation may draw out implicit knowledge in the schema—knowledge the subject has not attended to and/or not described linguistically before. For example, the simulation may draw out presymbolic knowledge embedded in analog tuning parameters of a motor schema to anticipate differences in the effort required to twist a long and short rod.

In other words, a hypothesis can be made, with initial grounding in data such as that in Episode 3, that the subject is going through a process wherein a general action schema assimilates the image of a particular object and produces expectations about its behavior in a subsequent dynamic image (simulation). The knowledge being used there is "embodied" in this sense. In the present cases, the action schema is equivalent to what, in natural language, one might call a "physical intuition".

Using the imagery indicators in Episode 3, one can now hypothesize that the subject generates an extreme case whose apparent role is to enhance the original physical intuition for the dependence of the needed twisting force on the length of the rod. This means that the extreme case works by enhancing the subject's ability to compare imagistic simulations for twisting a long rod and a short rod and that this comes from increasing the difference between the two images being compared and making that difference more detectable under inspection of the images.

Summary of Expert Findings

In summary, one can document the use of extreme cases in expert protocols. One can also point to evidence for the use of imagery and physical intuition schemas in such protocols. The latter finding led Clement (1994) to hypothesize that a process of imagistic simulation was involved in which the physical intuition is thought of as a perceptual motor action schema that can "manipulate" and anticipate predictions about an image of a physical system. One source of power in these extreme cases is explained by their ability to enhance imagistic simulations via a process called 'imagery enhancement' (cf. Clement, 2008, in press).

CLASSROOM CASE STUDIES

Examples from two classroom transcripts are presented. The examples from the first transcript, from a class on gravity, are of student-generated cases as well as teacher-generated cases, all of which appeared to lead to productive reasoning. The examples from the second transcript, from a class on normal forces, include examples that appeared to vary in their usefulness and point up both strengths and limitations in using this reasoning process in the classroom.

Gravity Lesson

The following is from a senior level high school physics class that had just finished a unit on density and was beginning a unit on gravity.[2] Among the common conceptions of students prior

[2] See Stephens and Clement (2006; submitted) for discussions of thought experiments and Gedanken experiments identified in this transcript.

to instruction is that one cause of gravity is the rotation of the Earth. The whole-class discussion from which the following excerpts were drawn comprised 42 minutes over the span of two days. The discussion began when the teacher introduced the topic of the causes of gravity with the following case, designed to elicit existing student conceptions.

US/Australia case.

The teacher drew a figure on the board (Fig. 3) and asked the class to vote on the following: "Compared to the United States, gravity in Australia is: a little less, equal, a little bit more." After the students had voted on voting sheets, the teacher asked, "Just what is it that causes gravity, anyway?" A very lively discussion followed in which the teacher played a role that was almost neutral, often merely restating student positions and asking for clarification.

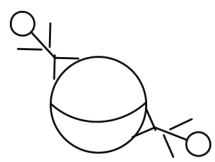

Figure 3. US/Australia Case.

Early in the class, some students suggested that the rotation of the Earth either causes gravity or is a major contribution to it. S5's response to this Rotating Globe variation sets the stage for an extreme case that followed.

40 S5: "Well, I just think that gravity has nothing to do with rotation, but maybe with rotation [*of the Earth*] like that guy [*referring to person standing on the Earth; see Fig. 3*] is trying to get thrown off the Earth. So he's getting pulled at the same rate but he's also getting pushed away."

When weighing oneself, the spinning of the Earth does reduce the reading on the scale very slightly, but many students have trouble imagining and understanding this effect and instead guess that spinning may be one of the causes of gravity.[3] In fact, even after S5's seemingly persuasive argument, the proponents of the spinning model of gravity appeared not to be convinced. For instance:

41 S6: "But the moon doesn't rotate and you can jump six feet on it."

46 S4: "But rotation. I'm almost positive that rotation does have something to do with gravity."

Whereupon another student proposes an extreme case variation on the Rotating Globe scenario, the Small Spinning Ball.

[3] Physicists consider centrifugal force to be a pseudo-force, but since this class is not yet near a level where they can understand that idea, the teacher does not broach this topic, allowing the students to react to S5's argument.

Small Spinning Ball extreme case variation.

49 S7: “Well in reference to rotation and gravitational force, I think of them as being two opposite forces because if you stand on— let's just imagine a ball floating in space you tape your feet to. And you start spinning the ball around, you're gonna feel like you're gonna be thrown off. But if it's a small ball, then the attraction between you and that little small mass is negligible so that you're just gonna [*inscribes rapid horizontal circles in the air with his forefinger*] feel the forces being spun around in a centrifugal force.”

The student seems intently focused on what he is saying. Why did he make the effort to recast what had already been said by S5 in Line 40? Does this contribute anything that S5’s statement had not? Both make the case that the pull of gravity and the force due to the rotation of the gravitating body are two different things and both use the example of someone standing on a spherical mass rotating in space.

The added element in this reasoning is that S7 took two variables to unusual values, one low and one high. The rotation of the spherical mass, which in the initial case of Line 40 produced one complete revolution every 24 hours, was taken here to a rapid spinning (to judge by the subject’s gesturing). Meanwhile, the pull of gravity was taken to a “negligible” amount. The result was a situation where the contrast between the effects of the gravitational force and the effects of the rotation was maximized (although the student’s terminology was imprecise). In this sense, the extreme case enhances the imagistic simulations involved. This *imagery enhancement* will be discussed more below.

This reasoning appears to play a role in the theory-construction taking place during this class discussion by helping to disconfirm spinning as a causal factor in gravity. We cannot know for sure the impact of this reasoning on the mental models of the other students. However, although one student continued to argue that rotation has something to do with gravity, the argument that it is the *only* cause or the *main* cause of gravity was not voiced again in the discussion by anyone after S7 introduced his extreme case.

There are 4 episodes of student-generated extreme cases in this transcript. However, if, as we have argued, this kind of non-formal reasoning can be involved in the theory construction efforts of experts and students, this suggests that it would be of benefit to be able to scaffold its use by students who are not spontaneously using such reasoning. There is evidence that this, also, occurred during the discussion.

North Pole/Equator extreme case variation.

Compare the following two statements, the first by S4, who had described spinning as an important causal factor in gravity; the second by the teacher. The teacher had just confirmed a student’s suggestion that any difference in the force of gravity at the bottom and the top of a local mountain would be minute, and any change in weight would probably not be measurable with the spring scales the class had available.

143 S4: “I think how far you are from the poles has more to do with it. I mean, I think if you're talking about really big distances then how far [garbled], but I think the Earth is pretty [*gesture indicating a round shape*] normal.”

144 T: “Now the other issue that you're bringing up that was kicked around some and not

resolved last time was that the gravity has to do with the Earth spinning, also is another issue that was mentioned. If that's the case, let's give it a little bit of thought about what [S4] is saying. If I were to stand at the North Pole, say the pole is here and I hold one hand on the pole, how long does it take me to spin around that pole?"

After some debate about details of this scenario, the class reached agreement that it would take one day and that the movement around the pole would be slow.

163 T: "My spin rate is not like I'm on a merry-go-round and I'm bumping. Thrown off into the sunset or something. Right? It's going to be pretty boring. This is almost like trying to watch the grass grow, right? At least for the speed effect you get out of it, right? Let me point, if I stand on the equator, however—"

164 S7: "You're going real fast."

The teacher has converted S4's phrase: "how far you are from the poles" into the extreme comparison of a person at the North Pole and a person at the equator, and students promptly began to reason about the comparison. In fact, this comparison continued as the topic of discussion over the next several minutes. During the course of this discussion, S4 appears to begin changing his mind as he runs the scenario as follows.

182 S4 *(off camera)*: "Ok, say that it's [*the rotation of the Earth*] throwing you. Then that still means that the top is still gonna be throwing you left and at the side [*at the equator*] [garbled]. So your weight's gonna be different."

Although part of his utterance is unclear, S4 appears to be conceding, at least for the sake of argument, that the rotation of the Earth could be "throwing you" rather than pulling the person in the direction of the Earth's surface. Furthermore, the rotation at the pole will be "throwing you left," presumably at a tangent rather than radially away from the Earth. Thus, the rotation at the pole would produce a different effect on one's "weight" than it would at the "side" of the Earth.

Although the student's earlier utterance in Line 143 had stated his belief—how far one is from the poles has an effect on gravity—it had not included a clear prediction. (An utterance shortly afterward indicated that he had still been thinking of rotation as "throwing you" at that point). The utterance of Line 182, after the introduction of the North Pole/Equator extreme case, though it is still vague, includes a novel, concrete prediction: the direction of rotation of the Earth will result in a difference between a person's weight at the North Pole and at the equator. It is doubtful that he would have been able to reason in this way with his original statement of Line 143. Thus, we hypothesize that the teacher's recasting of S4's statement has served to scaffold S4's thinking about his evolving model of gravity.

There is evidence here of conceptual change. Early in Day 1 of the discussion, S4 was the one who had first introduced rotation as an important variable to be considered, stating that he believed it "causes a lot of the main force of gravity." On Day 2, immediately before the teacher's introduction of the extreme case, S4 still appeared to hold this conception as he argued that how far one is from the poles has more to do with ones weight than ones distance from the center of the Earth. Soon after, he argued that spinning does not always try "to throw you away" but can help "hold." Now in Line 182, as the extreme case continues to be discussed, he has modified his scenario so that spinning "throws" rather than "holds" and is only one factor in a person's weight. Thus, the effect for S4 is now in the opposite direction and there is evidence that he has gone though some conceptual change since the beginning of the discussion of the extreme case.

S4 continues to refine his case:

186 S4: "How could that not have anything to do with it? If the Earth is trying to throw you off, in effect, at the equator, then it will kind of counteract the pull of the Earth on you. And at the North Pole it wasn't trying to throw you off and the Earth has more pull on you. Which means you'd weigh more. So it would change your weight."

At this point, the teacher notices two students in an intent side discussion and asks them to summarize what they are arguing about. S9 responds by re-running the extreme case variation. Again, we can ask what is added to the discussion by S9's merely re-running the same case S4 has already run.

189 S9: "What we were arguing about? Well I'm basically taking [S4's] position in that when the Earth spins, it seems logical to me, although [another S] says it's wrong, but it seems logical to me that there would be a force— say you're on the equator and you're going around, there's this greater force pushing you off the Earth than if you were on the pole and you're doing this little circle. [At the pole] it's just much less of a force throwing you that way. But if gravity is the same here [*gestures to indicate the side of an unseen object*], and gravity is the same here [*gestures to indicate the top of the same object*], it seems that you would weigh less here [*indicates the side*] because you're being thrown off more that way. Although you'd still stick to the Earth. You could still— I think you would weigh less."

Where S4 spoke of "the Earth pulling" and "the Earth trying to throw you off," S9 has clearly specified the forces involved (though one is a pseudo-force). He re-runs the extreme case, but specifies not only the variable that is changing with position (centrifugal effect), but also the variable that stays the same (gravitational force). His articulation has led to a more clear argument in favor of the prediction that one would weigh less at the equator than at the pole. This clarity, coupled with his reference to S4's argument, reinforces our impression that both students were equating the *reading* that would result on the spring scale and the *quantity* of weight; if one were to substitute "show a smaller reading on the spring scale" for "weigh less" in Line 189, S9's prediction would be accurate.

We hypothesize that the use of the extreme case of the difference in effects of gravity at the poles and equator led to productive reasoning that helped both S4 and S9 evaluate their explanatory models and helped S4 begin to let go of a persistent, common misconception about causes of gravity.

Book on Table Lesson

The following is from a senior level high school physics class that was beginning a unit on the concept of normal forces. A common conception prior to instruction is that inanimate objects cannot exert upward forces against gravity. In this lesson, the teacher wanted students to consider whether a table exerts an upward force on objects resting on its surface. The target model for the lesson was one in which objects exert normal forces that are equal and opposite to the weight of objects resting upon them. The whole lesson was structured around a series of bridging analogies (see the curriculum, Camp, et al. 1994; also Clement 1993); however the students came up with additional analog cases of their own. At one point, students appeared to be trying to take the sequence of analog cases to an extreme.

Table made of paper.

During discussion about whether a sturdy table exerts a force back upward against a book resting upon it (answer = yes; the 'normal force'), many students appeared to believe strongly that it could not. Most students did believe, however, that a spring would push back up against a hand that was compressing it. Students began considering a "warped" table as an object that might be able to move and to act a little like a spring:

S15: If you think about it, when the book is on the table, the table gets warped a little bit.

A number of students seemed to feel that a warped table would exert a force. A little later the teacher invited the students to consider an analog case: that of a book resting on "squishy foam stuff." Students began to discuss the possible similarities of this case with the hand-on-spring and book-on-table cases. But rather than considering the analog case, S15 suggested modifying the table:

S15: Wouldn't it, it make more sense if we build the table out of something pliable -

S3: Like plywood

T: Suppose we build the table out of something really cheap, I think I hear -

S(?): Yeah.

T: Really thin plywood, or -

S15: A piece of cardboard

T: or a piece of cardboard -

S15: Or a piece of paper

S(?) Bounty [referring to a brand of paper towel].

Soon after, S15 argues that the hard table will push back with the same amount of force as the bendy table, which is the scientifically accepted view. The bending of a paper table under a light load is a more exaggerated effect, and, we suggest, is easier to visualize than the bending of a sturdy table under the same load. We hypothesize that S15, with the support of the teacher, invented the extreme case of a thin table in order to make it easier to imagine the effect of a normally invisible variable at work within the book-table system (bending).

Some of the extreme cases posed by students during this discussion point to limitations as well as strengths that can be properties of this form of reasoning.

Digging to China (or: Not every extreme case makes a good argument).

One student argued that a rigid table does not have to move in order to hold the book up because "you could almost call it part of the ground." One could always make the book fall by digging a hole. "If you dig the hole, you're going to say the ground is moving too? So you dig the hole, and it falls again? You could dig all the way to China, it keeps falling." One way to interpret this argument is as follows: If you tell me the table bends and pushes up, then surely you won't admit this for the ground. And if you tell me it's true of the ground, then I can dig a hole and argue that the bedrock underground won't bend and push up. And if you tell me it's true of the bed rock, then I can dig through to China, and you will have nothing to point to that can push up.

Up to and including the bedrock argument, these extreme cases are legitimate (although ultimately incorrect from the physicist's point of view) attempts to find a case where it is obvious that the material cannot bend and push back.

The case of digging through to China is a beautifully imaginative attempt to make the normal force idea seem absurd. However, as a hyper-extreme case, it does stray outside the regime of the problem in question because it removes the central assumption that the book was resting on an object.

The other students did not follow up on this case and the discussion continued to move in the desired direction. The teacher was not so lucky with this next case, one that threatened to derail the discussion.

Platonic Table (or: Going beyond the domain of application).

A number of students seemed to feel that a warped table could exert an upward force upon an object resting upon it. However, one student wished to reason further and asked whether a perfectly rigid table would exert a force. When the teacher responded by referring to "those strong tables" in the back of the room, the student continued:

S4: But if we had an ideal table that did not move at all . . . then I don't see how it could be pushing up on the book.

This indicates the danger of an "extreme" extreme case taking the situation beyond the domain where it can apply. This extreme case could not move, by definition, and therefore could not act as a spring does. In bridging from a spring to springy objects (including the warped table) and on to rigid tables, the student had overshot the target model of a table that moves imperceptibly under a load and gone further, it appears, than the teacher had anticipated—to a table that was a Platonic ideal. This gave the student a new situation in which to run the model of (accelerated) movement-produces-force. Running the explanatory model in the new situation produced the prediction that the table would not be able to exert a force. Although this is, in fact, an extremely interesting idea, this student's extreme case had taken him outside of the real physical world where objects always deform under a force. There was nothing to prevent some students from drawing an unproductive inference, that a force was therefore not necessary to hold a book. Other students, however, correctly reasoned that, if the table were able to exert no force, the book would just fall down.

Later in the discussion, when the teacher tried to present an explanatory model of solids (as composed of atoms connected with spring-like-bonds), the Platonic table made an unwelcome re-appearance.

S: What about the Platonic table?

T: Well, um, what about the Platonic table? .... Um, personally, I think...it would still have to be made out of atoms.

S: But what about an ideal table….

T: Well, we have to decide, I think we ought to try to for the moment, to work with the notion of the table made out of atoms.

The teacher chose not to address the question of whether a table that could not move would be able to exert a force (since such a table could not exist), and instead used the opportunity to delineate the domain of application of the theory being constructed by suggesting that the students confine their reasoning to the physical world, in which tables are made out of atoms that are connected by bonds.

This was a student-introduced case, but it also suggests a cautionary note for teacher-introduced cases. Although this teacher pointed out the appropriate domain of application (the physical world, not a Platonic one), in our experience, teachers do not always clearly do this. Students can and do push cases into domains where the phenomena that result are not the phenomena under discussion. When choosing extreme cases to strengthen a lesson, we believe it is important at least to attempt to anticipate the domains into which students may be tempted to take them and be prepared to scaffold the discussion in the face of unexpected (and possibly quite creative) derailments.

In the following excerpt, several students pushed an extreme case beyond the domain of application but another student was able to address this shortcoming.

Elephant breaks the Table (or: Going beyond the domain of application).

Several students began discussing the case of an extremely heavy weight that could break the table. They debated whether breakage meant the table was unable to exert a force back against the object. S14 used the example of an elephant sitting on the table. Arguing that the table *can* push up, he said that the elephant can push down harder than the table can push up, just as a boat can move against a current by pushing harder than the current.

This argument did not convince S5. He replied, "See, but the table only has the power or whatever to just counter, it doesn't have enough to exceed and move in the other direction, but just counter, and as soon as (*the weight*) gets too great then the table collapses."

S15 responded by recasting S5's statement about collapse as an extreme case that goes too far. He argued that the existence of breakage does not disprove a prior occurrence of warpage.

S15: (*S5's*) idea is compatible with the warped table theory. The idea is that the elephant sitting on the table is too much for the material that the table is made out of, and it punctures the thing; it [*gesture*] warps it too much.

The shape depicted by the gesture was a curve much deeper than a table could normally form without breaking.

This enthusiastic discussion once again highlights both the strengths and potential weaknesses of extreme case arguments. On the one hand, they can tap into very clear, confident intuitions (prior knowledge schemas) that lead to a confident prediction for a clarifying case, accompanied by feelings of sense-making. However, the case of the weight extreme enough to break the table takes the situation into a totally different explanatory regime (here, fracturing phenomena) that was not intended to be under study. The two regimes are fundamentally different in terms of what is going on with the bonds inside the material. The broken table is not a suitable case for examining the forces that can result from warpage.

Had S15 not spoken up, the teacher could have taken the opportunity to delineate the domain of application of the theory being constructed (applying to elastic deformation but not to fracturing). However, S15 appeared to argue successfully that the existence of breakage is not inconsistent with a prior existence of warpage. By pushing the shape of the warped table to an extreme, S5 had transformed the warped table into the broken table, arguing that this was a different regime, and that the possibility of breakage was not evidence against the presence of warping.

In sum, the above conversation contains impressive creative reasoning reminiscent of the style of the dialogs of Galileo. It serves to illustrate the power and accessibility of extreme case reasoning, but it also illustrates a necessity for the teacher to be prepared to define the regimes of phenomena being modeled and to make sure students are aware when they leave one regime for another.

Imagery Evidence

For the classroom studies, the main imagery indicator used was depictive gesture. We categorized gestural episodes as Shape-indicating, Motion-indicating, or Force-indicating. ***Shape-indicating*** gestures [G-S] appear to depict a shape, and can be static gestures. These gestures can also involve movement, as when a subject moves her hands to indicate the shape of a globe. Either way, gestures that appear to depict a shape are taken as an indication of the presence of static mental imagery. ***Motion-indicating*** gestures [G-M] appear to indicate the motion of an object (it may be a point-object) and are taken as an indication of the presence of animated mental imagery. ***Force-indicating*** gestures [G-F] appear to indicate the action of a force and can be quite emphatic. These gestures are taken as an indication of the presence of dynamic/kinesthetic imagery; an example is shown in Figure 4. At times, an educated guess can be made from the appearance of the gesture alone as to whether it is intended to convey a motion or a force; however, we rely on the subject's use of *force terms* such as "pulling" or "throwing" as additional evidence for our choice between these two categories. (For a larger list of imagery indicators, see Clement 2008.)

Next, we briefly revisit student episodes discussed above and note the imagery indicators. The gesture category for each depictive gesture is indicated in square parenthesis at the point in the transcript where the speaker began the gesture.

Small Spinning Ball imagery (gravity transcript).

The Small Spinning Ball episode of S7 with gesture categories:

49 S7: "Well, in reference to rotation and gravitational force, I think of them as being two opposite forces because if you stand on— let's just [G-S] imagine a ball floating in space you tape your feet to. And you start spinning the ball around, you're gonna [G-F] **feel like you're gonna be** [G-F] **thrown off**. But if it's a small ball, then the attraction between you and that little small mass is negligible so that **you're just gonna** [G-F] **feel the forces** being spun around in a centrifugal force."

We hypothesize that this episode can be viewed as a student's effort to design a case that maximizes the potential of the rotating-globe scenario to evoke comprehension via kinesthetic imagery. It appears designed to help him and his classmates convincingly distinguish between the (felt) effects of rotation and the (felt) effects of the downward pull of gravity. His depictive gestures provide evidence for his own use of both animated and kinesthetic imagery throughout this episode. The phrases in bold are also regarded as evidence for kinesthetic imagery as he talks of feeling the forces; thus, there is considerable evidence for imagery in this episode.

This allows us to hypothesize that a function of the extreme case is *imagery enhancement*—wherein the case makes it easier to distinguish and discern the direction of forces. This is a different role for the extreme case than simply 'adding a new data point', as might be the case with inputting a zero value into a mathematical function to find an 'easy' data point.

North Pole/Equator imagery (gravity transcript).

As mentioned earlier, there was evidence for conceptual change for S4. His arguments on Day 1 of the discussion and early on Day 2 supported the idea that the rotation of the Earth is a part of gravity and is something that "can hold you." By the end of the second day, his arguments have changed. We take a look at imagery indicators in episodes that bracket this change. The first is a statement he made early in the first day of the discussion, in which he introduced rotation of the Earth as a variable to be considered. His gestures give evidence for dynamic imagery.

17 S4: I might be all messed up from reading too many science fiction novels but I thought that gravity [G-M]– when the Earth spins on its [G-S] axis the– I don't know [G-M] how but [G-M] somehow the fact that it spins causes a lot of the [G-F] **main force** of gravity. I agree with [another student] in that everything is [G-F] **pulling** on each other, but I think that that's not enough gravity. For instance, when you go to other planets that aren't spinning as fast, or that are smaller masses, there's not as much of a [G-F] **pull**.

At the beginning of the second day, when another student suggested that how far one is from the center of the Earth would have an effect on how much one would weigh, S4 indicates that he still believes the rotation of the Earth is an important factor in gravity:

143 S4: "I think how far you are from the poles has more to do with it. I mean, I think if you're talking about really big distances then how far [garbled], but I think the Earth is pretty [G-S] normal."

Lines 144-163 were discussed above. The teacher asks the class to "give a little bit of thought about what [S4] is saying" and recasts S4's statement as an extreme case. He invites the students to imagine a person standing right next to the North Pole and rotating around it with the Earth's daily rotation, and someone else standing on the equator. Soon after, S4's reasoning changes. We suggest that his depiction of the spatial orientation of the forces has also changed, as revealed through both words and gesture. The fact that in Line 182 he uses terms such as "throwing you left" indicates that he is now ready to consider rotational forces operating in a different direction.

182 S4 *(off camera)*: "Ok, say that it's [*the rotation of the Earth*] **throwing** you. Then that still means that the top is still gonna be **throwing you left** and at the side [*at the equator*] [garbled]. So your weight's gonna be different."

183 S5: "Yeah, but that has nothing to do with gravity."

184 S4 *(now on camera)*: "Why not? What if— the [G-F] Earth is **trying to throw you around** at the equator.

Figure 4. S4: "The Earth has more pull *on you."*

185 S5: “It's sort of like [turning] friction into a normal force and ….”

186 S4: “How could that not have anything to do with it? If the Earth is **trying to throw you off,** in effect, at the equator, then it will kind of [G-F] counteract the [G-F] **pull of the Earth on you**. And at the [*points*] North Pole it wasn't **trying to throw you off** and the Earth has [G-F] **more pull on you**. Which means you'd weigh more. So it would change your weight.”

This student appears to be putting considerable effort into both his words and his gestures. We suggest that one result of this is to make his imagined scenario easier for others to visualize; not only has the direction of the rotational forces changed since Line 17, but the placement, nature, and directions of the forces in this scenario are much clearer/less ambiguous than they were in Line 143, for example, immediately before the extreme case was introduced.

We hypothesize that one mechanism that made this possible was enhancement of the precision and clarity of the student’s imagery. It appears probable that this is due to the scaffolding of the teacher’s extreme case—considering effects at the pole and the equator—an apparently minor modification of S4’s own case. Though S4’s phrase “how far you are from the poles” does not differ from the North Pole/Equator variation in terms of which variables are involved, we suggest that taking the value of the variable to contrasting extrema provides clarity when visualizing the direction of the rotational and gravitational forces.

North Pole/Equator Re-Run imagery (gravity transcript).

S9 re-ran the North Pole/Equator extreme case variation in Line 189. The passage with gesture categories:

189 S9: What we were arguing about? Well I'm basically taking [S4's] position in that [G-M] when the Earth spins, it seems logical to me, although [S10] says it's wrong, but it seems logical to me that there would be a force— say you're on the [G-M] equator and you're

going around, there's this greater force [G-F] **pushing you** [G-F] off the Earth than [*points*] if you were on the [G-M] pole and you're doing this little circle. It's just much less of a [G-F] force **throwing you** that way. But if [*points*] gravity is the same here, and gravity is the same [*points*] here, it seems that you would weigh less [*points*] here because **you're being** [G-F] **thrown off** more that way. Although you'd still stick to the Earth. You could still— I think you would weigh less.

The presence of depictive gestures and kinesthetic imagery reports (in bold) again provides evidence for dynamic imagery in this case. Because S9's description is even clearer than S4's, we note the gradual improvement during the discussion of the ability to express the relevant distinctions. We hypothesize that there is a symbiotic relationship between precision in imagery and precision in verbal description—as the first improves, the second becomes possible, after which a cycle of each helping the other to improve can ensue (Clement, 2008).

Warped table imagery (book-on-table transcript).

The Warped table extreme case with gesture categories included:

S15: (*S5's*) idea is compatible with the warped table theory. The idea is that the [G-S] elephant sitting on the table is too much [G-S] for the material that the table is made out of, and it [G-F] **punctures** the thing; it [G-S] warps it too much.

"Puncture" is considered a force term and was accompanied by an emphatic gesture. The shape depicted by the final gesture was a deep curve, concave from above, much deeper than a table could normally form without breaking. The numerous gestures give evidence of the presence of visual imagery, and the force gesture accompanied by the force term *punctures* suggests the presence of kinesthetic imagery as the student appeared to embody the act of puncturing.

Tallying Student-Generated Extreme Cases

Four episodes in each of the classroom transcripts were identified as *student-generated extreme cases*, providing evidence that students can engage in spontaneous extreme case reasoning within the context of a classroom discussion. All but one of these eight student-generated episodes were accompanied by depictive gestures that appeared to depict either motion or force, indicating that dynamic imagery was involved.

Summary of Classroom Evidence

We found evidence in classroom case studies that students can reason with extreme cases proposed by others, can generate variations for those cases, and can invent their own extreme cases. There is also evidence that extreme case reasoning can be scaffolded. There is evidence for the use of both kinesthetic and visual imagery in these episodes. Finally, we noted evidence for conceptual change occurring after the introduction of an extreme case, on the part of a student who was able to use the case to sort out the directions of a force and a pseudo-force affecting ones weight reading on a scale.

The classroom extreme case exemplars presented here were part of two discussions in which students struggled to sort out the effects of different forces within their naïve models. In this way, it appears that this non-formal reasoning process has the potential to contribute to content goals in model-based science lessons. Scaffolding by the teacher may be important in order to help students home in on generating an extreme case, or occasionally to suggest directly an interesting extreme case for consideration. We have seen episodes where the latter can be done by the teacher without the teacher running or immediately making inferences from the case, and where students were able to take on these functions. It may also be important for the teacher to help define the regimes of the phenomena being modeled, and to help students become aware of when their extreme cases leave one regime for another.

SUMMARY OF FINDINGS

Our main findings are that it is possible to collect case study evidence indicating that:

1. Experts can generate creative test cases for extreme case reasoning when engaged in mental modeling, and
2. Experts can then use the process to reason about important steps in problem solving.
3. Students can generate creative test cases for extreme case reasoning when engaged in mental modeling, and
4. Students can then used the process to reason about important conceptual issues.
5. Teachers can scaffold extreme case reasoning during class discussions.
6. Experts and students can make use of mental imagery when engaged in extreme case reasoning and
7. At least some of this imagery is dynamic in nature.

In considering how the expert extreme cases yield predictions, we hypothesized that a process of imagistic simulation was involved in which a perceptual motor action schema "manipulates" and anticipates predictions about an image of a physical system. One source of power in these extreme cases is explained by their ability to enhance imagistic simulation comparisons in a process called 'imagery enhancement.' In one case, this included enhancement of the ability to discern the presence and directions for different (felt) forces. This is a different role for the extreme case than simply 'adding a new data point,' as might be the case with inputting a zero value to a mathematical function to find an 'easy' data point. Zietsman and Clement (1997) hypothesized a similar effect for extreme cases in teaching experiments on levers, but did not have the tools for analyzing imagery indicators.

IMPORT

Because it usually involves a simple transformation of one variable, we speculate that extreme cases may be one of the easier expert reasoning strategies for students and teachers to use. For example, it may be easier for students and teachers to generate relevant extreme cases than to find a useful analogy.

Some extreme cases are long on efficiency but short on providing understanding. So, for example the very wide spring is beautifully convincing and gives the answer in one quick step; however it gives relatively little insight into why the wide spring stretches more. This is a limitation that teachers should be aware of in using some extreme cases. Others like the spinning ball are perhaps more carefully designed and yield more insight. The case makes it easy to see the effects of the gravitational force and the centrifugal pseudo-force and to compare their directions and sizes—just what is needed here for progress on understanding. The case makes analysis easier, not just getting the answer easier. Also, as when the teacher modified a student case by casting it as an extreme case, such formulations may provide a way of enhancing the capabilities of student cases to elicit useful imagery. Thus, this reasoning strategy may be within reach of many physics students. The teacher did not have to run the case for the students; it is important to know that if the right case is generated or modified by the teacher, the students may be able to invest in it by running it themselves.

In one of the classroom transcripts, once one student generated an extreme case, other students began proposing extreme cases of their own in rapid succession. The fact that several students in each class generated their own extreme cases suggests this may be a natural reasoning process. In these transcripts, the process was used in connection with the development and refinement of mental models that involved unseen causes for phenomena. That students used this form of reasoning when trying to sort out the effects of different kinds of forces suggests that it can be used to promote process goals such as critical thinking.

Depictive gestures and other imagery indicators were associated with many of the student episodes of extreme case reasoning; this suggests that imagistic simulation was involved. Most of the student-generated cases had such indicators. A plausible explanation for this is that imagistic simulation is important for reasoning and sensemaking and that a role for extreme cases is to make imagistic simulation easier, clearer, or more possible for students. We called this role *imagery enhancement*.

A contribution to the development of learning theory is the hypothesis that extreme case reasoning can lead to more confident inferences by enhancing the use of imagistic simulations. Extreme cases are evidently not just a problem-solving heuristic; this process was used by experts and students for learning, building, and testing mental models that could provide causal explanations for phenomena.

For educators, the above evidence constitutes an initial "existence proof" that students can engage in extreme case reasoning via the use of dynamic mental imagery and that this can occur during model-based learning. We need to understand how to engage students in constructive reasoning processes for active learning. Teachers and researchers need to know that students can engage in certain types of powerful, non-deductive reasoning that fosters sensemaking. Further work is needed on the natural nonformal reasoning processes that students can do readily with some scaffolding, and the strengths and limitations of these processes.

For theorists, this study identifies a reasoning process that can contribute to theory construction in scientists, and suggests that it may involve imagery and simulation in a central way.

REFERENCES

Camp, C., Clement, J., Brown, D., Gonzalez, K., Kuduke, J. Minstrell, J., Schultz, K., Steinberg, M., Veneman, V., & Zietsman, A. (1994). *Preconceptions in Mechanics: Lessons Dealing With Conceptual Difficulties*. Dubuque, Iowa: Kendall Hunt.

Clement, J. (1993). Using bridging analogies and anchoring intuitions to deal with students' preconceptions in physics. *Journal of Research in Science Teaching, 30,* 1241.

Clement, J. (1994). Use of physical intuition and imagistic simulation in expert problem solving. In D. Tirosh (Ed.), *Implicit and explicit knowledge: An educational approach*. Norwood, NJ: Ablex Publishing Corp.

Clement, J. (2006). *Strategies for imagery use in expert protocols*. Proceedings of the NARST 2006 Annual Meeting, San Francisco, CA.

Clement, J. (2008). *Creative model construction in scientists and students: The role of analogy, imagery, and mental simulation*. Dordrecht: Springer.

Clement, J. (in press). The role of imagistic simulation in scientific thought experiments. *TOPICS in Cognitive Science*.

Darden, L. (1991). *Theory change in science: Strategies from Mendelian genetics*. New York: Oxford.

Driver, R. (1983). *The pupil as a scientist?* Milton Keynes: Open University Press.

Gentner, D. (2002). Psychology of mental models. In N. J. Smelser & P. B. Bates (Eds.), *International encyclopedia of the social and behavioral sciences*, Amsterdam: Elsevier Science.

Glynn, S. M., & Duit, R. (1995). Learning science meaningfully: Constructing conceptual models. In S. M. Glynn & R. Duit (Eds.), *Learning science in the schools: Research reforming practice* (pp. 3–33). Mahwah, NJ: Erlbaum.

Goldin-Meadow, S. (1999). The role of gesture in communication and thinking. *Trends in Cognitive Sciences 3*(11), 419-429.

Hammer, D. (1995). Student inquiry in a physics class discussion. *Cognition and Instruction*, *13*(3), 401–430.

Hegarty, M. (1992). Mental animation: Inferring motion from static diagrams of mechanical systems. *Journal of Experimental Psychology: Learning, Memory and Cognition, 18,* 1084.

Hegarty, M., Mayer, S., Kriz, S., & Keehner, M. (2005). The Role of Gestures in Mental Animation. *Spatial Cognition and Computation, 5*(4), 333–356.

Hostetter, A., & Alibali, M. (2004). On the tip of the mind; Gesture as a key to conceptualization. In K. Forbus, S. Gentner & T. Regier (Eds.), *Proceedings of the Twenty-Sixth Annual Conference of the Cognitive Science Society* (pp. 589-594). Mahwah, NJ: Erlbaum.

Iverson, J. M., & Goldin-Meadow, S. (1997). What's communication got to do with it? Gesture in children blind from birth. *Developmental Psychology, 33*, 453.

Iverson, J. M., & Goldin-Meadow,S. (1998). Why people gesture when they speak. *Nature, 396,* 228.

Lozano, S., & Tversky, B. (2005). *Gestures convey semantic content for self and others*. Twenty-Seventh Annual Conference of the Cognitive Science Society, Stresa, Italy.

Nersessian, N. (1995). Should physicists preach what they practice? Constructive modeling in doing and learning physics. *Science and Education, 4*, 203.

Reiner, M. and Gilbert, J. (2000). Epistemological resources for thought experimentation in science learning. *International Journal of Science Education*, *22*(5), 489–506.

Schmidt, R. A. (1982). *Motor control and learning*. Champaign, IL: Human Kinetics.

Vosniadou, S. (in press). *International handbook of research on conceptual change*. Amsterdam: Routledge.

Weld, D. (1990) Exaggeration. *Artificial Intelligence*, *43*(3), 311–368.

Zietsman A., & Clement, J. (1997). The role of extreme case reasoning in instruction for conceptual change. *Journal of the Learning Sciences, 6,* 61.