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# Analysis-Based Problem Solving: Making analysis and reasoning the focus of physics instruction

William J. Leonard, William J. Gerace, & Robert J. Dufresne

*Scientific Reasoning Research Institute and Department of Physics*
*University of Massachusetts, Amherst, MA 01003—4525 USA*

http://umperg.physics.umass.edu/

Walk into many classrooms today and look at the way the students are being taught, and you might think that the instructor is trying to pour knowledge into their students’ heads, as represented in Figure 1(a). The students are passively listening to a lecture, or watching a demonstration, and they might appear to be soaking up everything the teacher says.

Figure 1(b) (adapted from Van Heuvelen, 1992) is actually closer to what is really going on in many classrooms. The students are again passive, but little of the knowledge is retained. For instance, the teacher might be surprised when students don’t understand something after it is told to them once or twice. Or perhaps students consistently perform below expectations on quizzes, tests, and exams. This representation is somewhat flawed, however, because it assumes that efficient communication is occurring in the classroom and it blames the students for any failures to understand.

(a) (b) (c)

**Figure 1: Three views of common classroom practices.** (a) The teacher is pouring knowledge into the student’s head. (b) The teacher is trying to pour knowledge into the student’s head, but the knowledge just spills out again. (c) The teacher is trying to pour knowledge into the student’s head, but the inverted funnel prevents most of the knowledge from going in. The student is using her finger to try to keep the knowledge from spilling out, but she fails.

Figure 1(c) shows our view of common instructional practices, and there are two features worth noting. First, the finger in the ear means that students are trying to retain the knowledge but, lacking the skills needed to do so, fail. Listening to lectures, taking notes and studying them, reading textbooks, memorizing formulas, doing problems, etc. are not sufficient for learning or understanding the desired material. Second, the inverted funnel means that common modes of instruction create a mismatch between the students and the teacher, which can cause little meaningful communication to occur. Students do not always understand what the teacher is saying, while the teacher is getting no feedback about what the students do and do not understand. Communication occurs in only one direction—from the teacher to the students—so the teacher might not even be aware that there is confusion.

Learning science is particularly difficult for many students. In the pursuit to cover topics, many teachers revert to modes of instruction that are efficient for delivery (i.e., lecturing and then assigning lots of problems), rather than efficient for learning. Most teachers value problem solving, and they generally think that if students can solve problems, then they must understand the material. The irony of physics instruction is that although students solve lots of problems they do not generally develop good problem-solving skills. Rather, solving lots of problems encourages and reinforces formulaic approaches and superficial learning. Further, success in solving problems is not usually a good measure of conceptual understanding.

Other approaches seem to sacrifice problem solving for deep understanding. These modes of instruction focus so much attention on concepts that teaching and assessing conceptual understanding becomes a goal rather than a means to an end. And what, therefore, is the value and purpose of conceptual understanding? We believe that the value of a deep understanding of concepts and principles is to be able to apply knowledge flexibly to solve unfamiliar problems, which is why problem-solving ability without conceptual understanding is not valued by most instructors.

Therefore, excessive emphasis on either problem solving or conceptual understanding is undesirable. But both are valuable facets of physics instruction, so how can both be encouraged without going too far?

We believe that both deep understanding and proficient problem solving stem from being able to analyze situations conceptually. Thus, analysis becomes the bridge that allows concepts to be useful for problem solving. Our approach, called *analysis-based problem solving*, shifts the focus of instruction from either lecturing and problem solving or misconceptions and conceptual understanding to activities that target beneficial cognitive processes.

Learning science is like climbing a ladder. To get to the top of the ladder, you need to have both legs working together. One leg supports you as the other leg raises you up to the next rung. In this same way, conceptual analysis and problem solving work together, each one supporting and improving the other.

To help students develop both conceptual-analysis skills and problem-solving skills at the same time, we take students through a sequence of learning experiences. First, students explore their pre-existing notions so that they do not interfere with scientific concepts. Second, students sharpen, link, and interrelate concepts, creating a rich network of ideas that helps them to understand and remember concepts. Third, students learn how to use concepts to analyze and to reason about common situations, which makes it possible for students to solve interesting, unusual, or complex problems. Fourth, students develop general problem-solving skills based on expert-like strategies using principles, rather than novice-like approaches using superficial features. Fifth, students organize and prioritize their knowledge so that it is particularly useful for both problem solving and analysis. A consequence of this approach is that students can solve more difficult problems than students who follow a traditional approach, and they usually improve their reasoning skills and develop a deeper understanding of concepts and principles. Concepts, rather than equations, become the language students use to learn and to do physics.

For each of the five types of learning experiences described in the paragraph above, we will present specific research results and pedagogy that are relevant for understanding it. Then we will recommend some instructional modes and classroom practices that we think are particularly useful for creating rich educational experiences for students. These modes and practices have been used to create an extensive set of physics instructional materials (Leonard et al., 1999, 2000, 2001).

## I. Exploring Students' Existing Concepts

**Research results.** Students come into our classrooms with deeply held ideas of how the world operates, many of which survive despite instruction, and often they co-exist side-by-side with “scientific” ideas. These alternative conceptions generally arise because the human mind is constantly trying to account for and cope with experiences, and so students already have a conceptual framework when they begin a science course, though this framework might seem inconsistent to an expert or a teacher. Prior conceptions have been shown, in many instances, to impede learning of more formal frameworks. Prior conceptions can be difficult to uproot and are often retained even after a concerted effort has been made (by instructors) to get rid of them. (For reviews and bibliographies, see Maloney, 1992, McDermott, 1984, McDermott y Redish, 1999, Mestre, 1991, 1994, Pfundt y Duit, 1991.) Research also indicates that each person's organizational scheme is unique, and that people often have difficulty using or fully appreciating another person's scheme. The best students often seek patterns on their own, but most students need to have patterns pointed out to them explicitly (Bassok y Holyoak, 1989; Gick y Holyoak, 1980, 1983, 1987).

Classroom observations at all levels indicate that teachers seldom take into account the conceptual knowledge previously constructed by students. Further, students' ideas, predictions, and explanations of science phenomena are not probed to determine whether the concepts being taught are in conflict with students' prior notions (Hewson et al., 1995; Resnick, 1983). Research results strongly suggest two additional findings: (1) teachers are generally not aware of the knowledge state of their students, and (2) traditional assessment practices do not measure conceptual understanding (Chi y Glaser, 1981; Harmon y Mungal, 1992; Hestenes y Wells, 1992; Hestenes et al., 1992; Kulm y Stuessy, 1991).

**Pedagogy.** *Constructivism* is a philosophy that states simply that all knowledge is constructed as a result of cognitive processes within the human mind. It rejects the idea that knowledge is a representation of an external, observer-independent reality. (Science, of course, presumes the existence of an external reality and seeks to describe and explain its nature and behavior.)

The premises of constructivism are:

- *Knowledge is constructed, not transmitted*. Experiences must be interpreted and processed by each individual. Two people cannot exchange knowledge as though it were mere information.
- *Prior knowledge impacts learning*. Existing cognitive frameworks determine what people notice, how they interpret what they notice, and how they construct new knowledge. Two people might have the same experience, but they can have different interpretations of it.
- *Initial understanding is local, not global*. New ideas are necessarily introduced and understood only in a limited context. When an idea is first introduced, it can be difficult for someone to know what features of the situation are most relevant for understanding it. Later, when the idea has been explored in a variety of contexts, it is often easier to perceive the intended pattern, and understanding is generally wider.
- *Building useful knowledge structures requires effortful and purposeful activity*. Meaningful learning requires active and thoughtful engagement.

For pedagogic purposes, the premises of constructivism may be rephrased as follows:

- Students have an established world view, formed by years of prior experience and learning.
- Even as it evolves, a student's world view filters all experiences and affects all interpretations of subsequent observations.

- Students are emotionally attached to their world views and will not give them up easily.
- Challenging, revising, and restructuring one's world view requires much effort.

The first step in the learning process, therefore, is to make teachers and students aware of their world views. The more teachers know about students' individual conceptual frameworks, the better they can reveal the limitations of those frameworks, and the more likely they will be able to induce students to re-think and re-formulate their own world views (Anderson, 1987; DiSessa, 1988; von Glasersfeld, 1989, 1992; Resnick, 1983, 1987; Ritchie et al., 1997; Schauble, 1990). Learners should be self-aware, and they should be fully engaged in the learning process. They can deliberately seek supplemental learning experiences, and they can be very effective at modifying their own world views (Brown y Clement, 1989; Camp y Clement, 1994; Clement, 1993; Mestre et al., 1997; Wenk et al., 1997).

**Instructional Modes.** Some useful modes for exploring students' existing concepts are:

- *Use Predict & Show (inadequacy of old model)*. When students predict the outcome of a demonstration or experiment, they are more committed to the activity and they are less likely to withhold judgment, i.e., less likely to wait for the teacher to interpret it for them. By predicting an outcome, students often reveal the features of a situation they are focused on and show what features they consider most relevant for understanding it. Although each student's world view often remains implicit during this instructional mode, an incorrect prediction can be ideal for demonstrating that their model has limited applicability and for preparing them for learning.
- *Explain (draw, describe, discuss)*. When students explain their reasoning for an answer, draw a picture of something, describe an observation, or discuss a demonstration, they are forced to use and make explicit whatever models they have for organizing their experiences, and they are often forced to use concepts in their explanations or descriptions. This instructional mode promotes self-awareness. Students cannot change their world views effectively unless they are aware of them.
- *Communicate about the learning process*. Students need to know that each of them has a unique perspective and that sometimes it is not self-consistent and can prevent them from learning efficiently and understanding deeply. They need to become active participants in the whole educational endeavor, from exploring existing concepts to structuring knowledge. Communication about learning is one way to help them.

**Classroom Practices.** There are many ways that instructors can help students to reveal and confront their world views. These include:

- *Have students use their own models to answer open-ended questions*. Good questions are often simple questions within a familiar context. Good questions reveal the limitations of students' models and motivate them to seek new conceptions. The answers inform the teacher and promote self-awareness among students.
- *Tell students that they should not seek "right" answers; answers should be whatever each student believes is true*. The hard part is creating a classroom atmosphere or environment in which students are comfortable taking risks and exhibiting their own thoughts, rather than feeling forced to give teachers what they (the teachers) want. In a constructive atmosphere, students are more likely to engage in the process of learning and understanding. When questions involve common experiences, answers can often be tested with simple experiments or demonstrations, and science becomes integrated with students' everyday lives.
- *Use small groups*. Every student's world view does not need to be confronted by the instructor; another student can be as effective as the teacher—sometimes even more so—at uncovering inconsistencies in a world view. Small group discussions help students refine their explanations of the reasoning used to answer questions, and are usually less threatening than whole-class discussions. Using small groups makes students aware of diverse perspectives and promotes the idea that what everyone thinks is important. It also helps students refine definitions. Perhaps most importantly, using small groups integrates language with science and experience.

## II. Honing and Clustering Concepts

**Research results.** Table I summarizes the main differences between the knowledge characteristics of experts and novices (Chi y Glaser, 1981; Glaser, 1992; Larkin, 1979).

**Pedagogy.** We have organized these results into a representation of the expert's knowledge store, as shown in Figure 2.

Domain knowledge has been divided into three general categories:

- *Operational and Procedural Knowledge*, such as the definition of kinetic energy, how to draw a free-body diagram, and how to find the normal force.
- *Problem-State Knowledge*, i.e., the features of a problem or situation used to characterize it. These can be surface characteristics, such as an inclined plane or a spring, or concepts and principles, such as that energy is conserved.
- *Conceptual Knowledge*, such as force, mass, acceleration, kinetic energy, etc.

The expert has a rich, hierarchical (prioritized) clustering of conceptual knowledge. Concepts are linked to many different operations, procedures, and problem situations, thereby refining and sharpening the meanings of the concepts. Experts use concepts to characterize problem situations and they also use concepts to judge the appropriateness and applicability of equations, operations, and procedures.

The novice has a very different knowledge structure, as shown in Figure 3. Unlike experts, novices generally have a poor clustering of concepts. Many links are inappropriate; others are non-existent. Some of the inappropriate links are extremely strong, which can lead to misconceptions. Many novices are familiar with or have memorized a large number of equations, but they often remember them incorrectly or need to look them up in order to use them. They have been taught operations

**Table I: A comparison of the knowledge characteristics of experts and novices.**

| Expert | Novice |
|---|---|
| • Large store of domain-specific knowledge | • Sparse knowledge set |
| • Knowledge richly interconnected | • Knowledge mostly disconnected and amorphous |
| • Knowledge hierarchically structured | • Knowledge stored chronologically |
| • Integrated multiple representations | • Poorly formed and unrelated representations |
| • Good recall | • Poor recall |

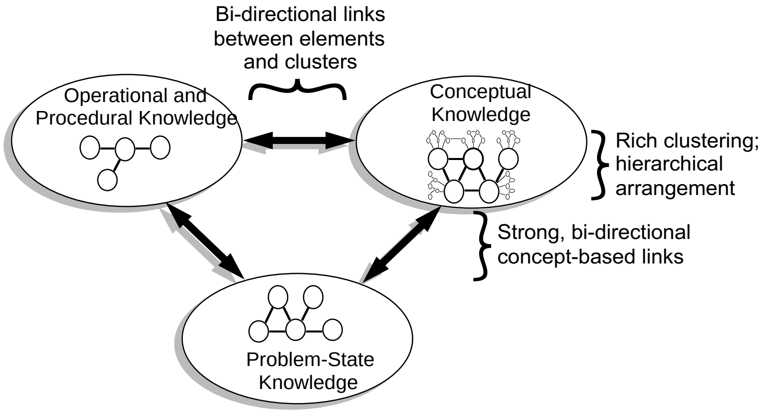

**Figure 2: A representation of the expertÕs knowledge store.**

and procedures, but they are not yet proficient at using them and therefore often avoid using them. The links between equations and problem situations are relatively strong, but are based largely on the quantities (given and unknown) mentioned in the problem or situation.

In our view, one goal of instruction should be to help students develop a rich, concept-based structure of knowledge. To do this, students must learn how to hone (sharpen) and cluster (interrelate) ideas (Minstrell, 1992).

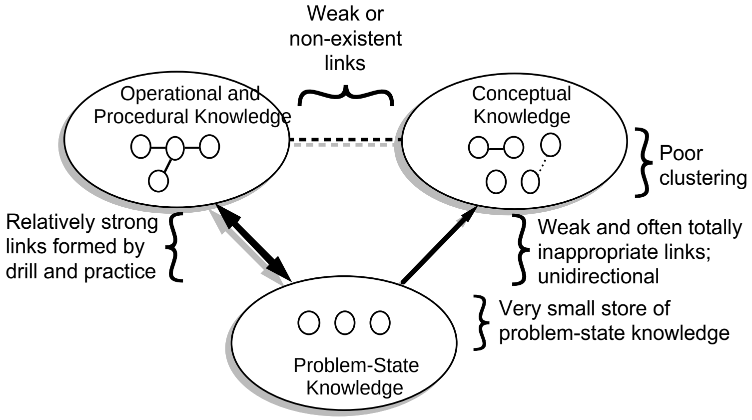

**Figure 3: A representation of the noviceÕs knowledge store.**

**Instructional Modes.** Here are some modes relevant for helping students hone and cluster concepts:

- *Use Multiple Representations*. A representation may be linguistic, abstract, verbal, symbolic, experiential, pictorial, physical, or graphical. Deep understanding of any concept requires many representations, yet students often think that one representation (the algebraic) is sufficient. Students also tend not to interrelate representations, which often means that their abstract physics ideas are not well connected to real-world experiences. Using different representations for the same knowledge, and having students translate between representations, helps students to interrelate ideas and to relate ideas to personal experience. For instance, write an equation on the blackboard and have students read it back to you (i.e., translate from the algebraic to the verbal representation). We especially like to use graphs because they are abstract, like equations, but can be understood qualitatively, like diagrams or pictures.
- *Explore Extended Contexts*. Initial understanding is necessarily limited by the context in which it is first introduced. The human mind naturally seeks patterns and tends to generalize using those features that are most noticed. Students tend to focus on surface features and often generalize incorrectly as a result. Students also cannot easily re-evaluate their generalizations. Investigating a broad set of problem situations helps students to refine and to abstract concepts; it avoids inappropriate or oversimplified generalizations. Students are more likely to use relevant features and to ignore irrelevant features after they have explored a range of contexts.
- *Use Compare & Contrast*. Like Extended Contexts, the goal of Compare & Contrast is the interrelation of knowledge. The difference is that while they are Comparing & Contrasting situations, students are required to look explicitly for distinctions and commonalities between situations.
- *Explain (describe, discuss, define)*. When students explain (and describe, etc.) their reasoning, they reveal the features they are using to organize their ideas. Explaining also helps students create connections between ideas.

**Classroom Practices.** The goal of these activities is to help students to refine and to generalize their definitions of concepts, and to relate new ideas to ideas already learned. The following classroom practices should make this more likely:

- *Use as many different representations as possible for the same concept*. Using different representations helps students sharpen their understanding and provides alternatives for thinking about a concept. This can be especially important when a particular concept is needed to understand a more complex concept.

- *Make sure the first few examples of something are the same* <u>*only*</u> *in the feature (or features) that is relevant for understanding it.* Students are likely to notice many similarities between two situations. We cannot guarantee that they will notice what is relevant and ignore what is not. (And telling them what to notice and what to ignore is not sufficient!) For instance, many students believe that the normal force always points vertically upward, because all the examples they have seen share this feature. By taking greater care in choosing the first two or three examples of something, teachers can help students avoid oversimplified generalizations and confusion.
- *Ask questions that probe the boundaries of students' knowledge.* Learning occurs at the periphery of understanding when students attempt to use and relate partially formed ideas and work outwards from a core of well understood ideas. Learning cannot occur in the darkness and confusion of poorly formed ideas. For example, have students compare similar situations: is [something] the same or different? why? Change something about a situation; what else changes? Have students provide examples (preferably from their everyday experiences) in which the concept is or is not manifest.

## III. Developing Analysis and Reasoning Skills

**Research Results.** Most beginning physics students do not appreciate the value of a conceptual analysis as part of solving problems. Instead, novices usually perform means–ends analysis. They focus on equations and start manipulating them in an attempt to isolate the desired unknown, often inserting numerical values from the very beginning of the process. Novices are distracted by the goal of determining the value of the desired quantity. Within this mental state, students often suffer from *cognitive overload*. They are so focused on answers that they have no mental resources left to think about problem solving (Dufresne et al., 1992; Larkin, 1981, 1983).

**Pedagogy.** The manner by which students learn is itself learned. What students know determines how they engage in problem-solving activities, and how they engage in problem-solving activities determines what they learn. Their approach to problem solving has been reinforced by years of rote learning, memorization, and regurgitation. Continuing to assign lots of problems does little to break this cycle. In fact, doing traditional problems can reinforce superficial attitudes and discourage students from desiring to understand (Brown et al., 1989; Touger et al., 1995).

One solution is to structure problem-solving activities to shift the focus of students' attention away from getting an answer, and to communicate with students about learning so that they are looking for alternative patterns and explanations. For example, students must be made aware of their learning habits. Before students can use concepts to solve problems, they must learn how to use concepts to analyze situations and reason about them (Dufresne et al., 1997).

**Instructional Modes.** Some modes useful for developing analysis and reasoning skills are:

- *Use Multiple Representations*. The essence of effective reasoning is finding the representation in which the result is most obvious. (For example, in many situations free-body diagrams are useful for comparing the magnitudes of forces. Also, a graph can be useful for reasoning toward an answer.) When students consider different representations and use them to analyze situations, they can improve their critical thinking skills, which can lead to improved problem-solving proficiency.
- *Use Compare & Contrast*. It usually takes less time to compare two quantities or two situations than to compute a single quantity. Often, we can describe a situation without specifying enough information to determine unknown quantities, yet there is still enough information to make a comparison. (For example, consider a disk and a sphere having the same mass and radius rolling at the same speed along a horizontal surface. Upon encountering an incline, which would reach a higher maximum height?)

- *Explain (summarize, discuss, listen, debate, argue).* Good critical thinking skills can be developed and honed by having students explain their reasoning and by having them debate issues. Listening requires analysis and processing. Presenting counter-arguments to someone's line of reasoning further knits the structure of knowledge. Reasoning necessarily involves concepts, so concepts become the vocabulary of explanations, eventually making qualitative analysis a viable tool for problem solving.

**Classroom Practices.** The following suggestions can help students focus their attention on concepts as being useful for understanding physical situations.

- *Use goal-free activities.* To reduce cognitive load and encourage reflection and deep thinking, students need to work on questions that do not require a numerical result. This makes it more likely that they will use concepts. Ask questions that can only be answered using concepts, or at least, can be answered most easily using concepts. Equations might be used, but usually they are not manipulated in order to solve for an unknown. For example, a definition might be used to determine how one quantity is related to another.
- *Direct students' attention to the features of a situation most relevant for understanding it.* For example, in one situation, energy might be the most relevant for understanding it, while in a similar one, momentum might be more relevant. Students might be focused on the surface similarities, and might not realize that different concepts are used. Analyzing situations encourages using scientific concepts and principles to organize knowledge.
- *Use familiar or simple situations, or use the same situation to ask many different types of questions.* Students invest a lot of mental resources in processing and storing the context in which a question is posed. When the situation is unfamiliar or complicated, there may be few resources left for students to analyze it. By using familiar or relatively simple situations, cognitive load is reduced. By re-using situations in many different problems and questions, students learn that the same situation can be analyzed and understood using many different concepts and principles, and they learn that surface features are not always useful for organizing knowledge.

## IV. Developing Problem-Solving Skills

**Research Results.** The problem-solving behaviors of experts and novices are very different, as summarized in Table II (Chi et al., 1981; Glaser, 1992; Hardiman et al., 1989; Larkin et al., 1980a, 1980b).

**Pedagogy.** Figure 4 shows a representation of *strategic knowledge*. Strategic knowledge links knowledge of problem situations, equations, operations, and procedures into a strategic element that guides the entire problem-solving process. Based on a conceptual analysis, decisions are made concerning what ideas should be focused on and which should be ignored. Students will not build strategic knowledge elements on their own. They must engage in structured problem-solving activities and they must reflect on the problem-solving process (Dufresne et al., 1992).

Proficient problem solving is mostly about making choices—what to focus on, what principle to apply, what representation to use, what to ignore—yet most students have (at best) one way of solving any particular problem. Choices necessarily involve concepts, because it is concepts that are used to make comparisons. The more we can encourage students to perform qualitative analyses before solving problems, the more they will improve their problem-solving proficiency.

**Instructional Modes.** The following modes can be used to help students develop forward-looking problem-solving approaches and abandon backward-looking means–ends approaches:

- *Use Classify & Categorize.* This is similar to Compare & Contrast, except that the focus here is on sorting ideas or problems, and on labeling the resulting categories. Students must practice creating and

**Table II: A comparison of the problem-solving characteristics of experts and novices.**

| Expert | Novice |
|---|---|
| • Conceptual knowledge impacts problem solving | • Problem solving largely independent of concepts |
| • Often performs qualitative analysis, especially when stuck | • Usually manipulates equations |
| • Uses forward-looking concept-based strategies | • Uses backward-looking means–ends techniques |
| • Has a variety of methods for getting unstuck | • Cannot usually get unstuck without outside help |
| • Is able to think about problem solving while problem solving | • Solving problems uses all available mental resources |
| • Is able to check answer using an alternative method | • Often has only one way of solving a problem |

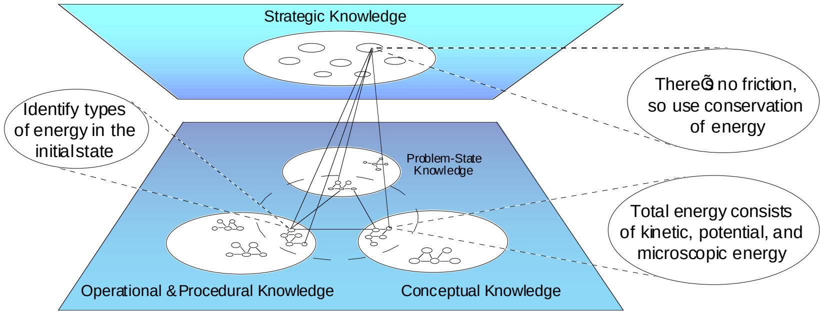

Figure 4: A representation of strategic knowledge.

recognizing classification systems. When students sort items, choose names for their categories, and explain their systems, we increase the likelihood that they will use concepts to organize their knowledge.

- *Generate Multiple Solutions*. When students solve the same problem using different approaches, they learn to prioritize the approaches. For example, the algebraic representation is not always the most useful one for solving a problem, but students are generally not convinced of this.
- *Plan, justify, and strategize*. Very few relationships in physics are always valid. Many are derived or defined for a particular set of circumstances. (For example, the definition of kinetic energy as $^{1}/_{2}mv^{2}$ is true only for a point object or when the object is both rigid and not rotating.) Most students are not aware of the assumptions, conditions, and circumstances that make a particular equation, operation, or procedure applicable. By having students plan their approach (without actually solving the problem), justify their approach, or develop a strategy, they learn the value of concepts and conceptual analysis for problem solving (Leonard et al., 1996).

**Classroom Practices.** Here are some ideas to help teachers implement structured problem-solving activities.

- *Choose problems that require a conceptual analysis to solve*. Traditional problems are usually solved by applying one or more previously derived equations or by repeating standard procedures by rote. Understanding is not required for traditional problems. However, when problems cannot be solved using a formulaic approach, students are more likely to realize that the most efficient method is by using concepts. This does not mean that the problems need to be difficult (though students will find them difficult at first). In fact, the best problems use simple or familiar situations and are relatively easy to solve with an analysis-based approach.
- *Have students explain how they would solve a problem*. In the amount of time it takes most students to solve a single problem, students can explain how they would solve a number of problems. This activity shows students (and the teacher!) what the students are focused on while problem solving, which impacts later discussions and allows the teacher to plan suitable interventions.
- *Have students solve the same problem using different approaches*. For example, many problems can be solved using either Newton's laws or Momentum Conservation. Solving the same problem a second time using a different principle helps students learn new material and promotes the comparison of problem-solving methods.

## V. Structuring Knowledge in Memory

**Research Results.** In a series of PERG studies, students solved problems by answering a sequence of questions arranged hierarchically. For instance, they were first asked, “Which of the following would you use to solve this problem? Kinematics / Newton's Laws, Work–Energy / Conservation of Energy, Impulse–Momentum / Conservation of Momentum, or Angular Impulse–Angular Momentum / Conservation of Angular Momentum.” Depending on their answer to the first question, they would answer increasingly focused questions about the problem. Control groups solved the same problems and spent the same amount of time solving them as the test group. Two relevant results emerged from these studies: Students constrained to solve problems hierarchically (1) were more likely to get the problems correct; and (2) were more likely to sort problems according to which principle was used to solve them (Dufresne et al., 1992; Mestre et al., 1993).

**Pedagogy.** Traditional problem-solving tasks do not help students develop useful problem-solving skills. We believe the reason for this is that traditional problems do not stimulate beneficial cognitive processes. The core of our cognitive framework is this:

- Particular types of knowledge and knowledge structures are needed for proficient problem solving and deep understanding.
- Particular types of cognitive processes are required for the acquisition of conceptual knowledge and the building of useful knowledge structures.
- Activities should be designed to encourage desirable cognitive processes.

The ultimate goal remains proficiency at solving problems, but equally important goals are developing deep understanding of physical situations and the ability to analyze new or unfamiliar situations using physical principles. To accomplish these goals, the main focus of most activities is shifted away from problem solving toward cognition. We cannot guarantee that beneficial cognitive processes will occur, but we can make them more likely to occur. This, we believe, should be the focus of instruction (Dufresne et al., 1997; Leonard y Gerace, 1996).

**Instructional Modes.** These modes can help students structure their knowledge for deep understanding and for efficient and effective problem solving.

- *Make forward and backward references*. Concepts require a long time and much experience to become fully formed. You cannot wait for students to completely learn one topic or idea before moving on to the next. By making forward references to material to be covered, you prepare the student for new material. By making backward references, you associate new material with established or partially established material, making knowledge interwoven and interconnected, rather than linear or chronological. By making forward and backward references, students can construct many pathways to the same ideas, thereby making knowledge more easily accessible for discussions, analysis, reasoning, and problem solving. Students can also improve their performance on tests, quizzes, and projects.
- *Use Classify & Categorize*. When students classify and categorize ideas, we can increase the sophistication of the ideas used to think about physics and physical situations. In particular, students are more likely to use physical principles and laws to sort problems and problem situations. The question is, "Are your students aware of the categories that <u>you</u> believe are useful for organizing physics ideas?" If so, then they should be able to notice when two problems are solved using the same principle, even if the two problems look very different. If not, then using Classify & Categorize can reveal how students are sorting problems, and can lead to useful discussions about other possible organizational systems.
- *Reflect (evaluate, integrate, extend, generalize, etc.)*. After most activities, students benefit from reflecting on what they've just done. What patterns have students perceived in the questions, situations, or problems presented? How will students approach similar situations in the future? What difficulties were encountered? What caused the difficulties? How will they overcome them in the future? How would they apply the ideas to... [a specified situation]? Can they connect the ideas to "real-world" events and situations? What general principles may be extracted from the learning experience? What have the students learned?

  Learning experiences often give students the necessary pieces of the "how-to-do-physics" puzzle, but many students will not attempt to fit the pieces together unless you ask them specifically to do it and give them time to do it. Reflection helps students structure their knowledge <u>as they are learning it</u>.

- *Communicate about the learning process*. To learn physics (and many other subjects!) students must become self-invested in the learning process; they must become more self-aware and more self-motivated; they must know why learning physics is useful and important. These issues are handled by communicating <u>about</u> learning. Do students know how they learn best? Have they ever thought about what their greatest strengths (or greatest weaknesses) are? Do they know what the purpose of a particular activity is? Do they understand why concepts are so important? Do they know what is meant by "structured" knowledge? Do they know why structuring knowledge is useful? Communicating about learning helps motivate students and helps keep them engaged in the learning process.

**Classroom Practices.** The following practices will help teachers help students structure their knowledge.

- *Give students many opportunities for reflection*. Reflection is an activity that teachers and other professionals usually perform on their own, even while engaged in another activity. (For example, a teacher can usually solve a problem and evaluate the solution process at the same time.) Most students cannot reflect on problem solving (or any other activity) *while* doing it and they will not reflect on their own; they must be given time specifically put aside for reflection, and they must be given questions specifically designed to help them do it. There are many contexts students can reflect upon: the ideas raised in an activity or discussion they've just completed, problem solving, their learning styles, and the entire learning process, just to name a few.
- *Give students the time needed to think about and discuss the interrelation and prioritization of ideas*. Structured knowledge is richly interconnected and organized hierarchically, with the most important ideas serving as umbrella concepts for less important ideas. Structured knowledge leads to efficient and effective problem solving, because concepts can be used to determine the applicability and appropriateness of equations, operations, and procedures. Structured knowledge also leads to deeper understanding.
- *Give problems in which the surface features may be misleading*. When the surface features of a problem suggest an approach that is different from the most efficient or effective approach, students learn that concepts are useful for organizing ideas. The problems do not need to be difficult, however, and the best ones involve situations that are easy to describe and easy for students to understand. The goal is to design problems that require a conceptual analysis to solve.

## Conclusion

For many students, trying to learn science is a downward spiral of confusion and failure. Classroom practices do not meet their needs, nor do practices seem to take into account their knowledge and skills. Every teacher has high expectations for their students. They want students not only to understand the material, but also to develop higher order thinking skills, such as analysis, reasoning, and problem solving. Unfortunately, the day-to-day practices in most classrooms encourage students to learn only superficial ideas and low level skills. Without a strong conceptual foundation and without analysis, reasoning, and other skills, students will continue to adopt superficial and formulaic approaches to problem solving.

Analysis-based problem solving addresses these issues. Taking into account students' prior notions enables them to distinguish scientific definitions of concepts from everyday definitions. Having them analyze physical situations helps them to relate concepts to each other, and improves their conceptual understanding even more. Having them reason about physical situations helps them to solve interesting problems. Encouraging them to reflect on prior activities helps them to structure their knowledge. This is the sequence of learning experiences by which students acquire the knowledge, the skills, and the opportunities to succeed. By using analysis and reasoning as a bridge between deep, conceptual understanding and proficient problem solving, we can create a classroom environment where learning flourishes.

This does not mean that equations are not important or useful. It means only that equations are not the most important or useful part of doing physics. Equations are needed, but only near the end of the problem-solving process, when principles, laws, and definitions are applied.

It also does not mean that concepts are not important. Rather, it means that conceptual understanding alone is not sufficient to promote the problem-solving skills we, as instructors and educational researchers, desire and value.

Our image of effective classroom practice is very different from the images shown before. Communication is two-way, with the student communicating with the teacher as much, if not more, than the teacher is communicating with the student. The student is an active participant in the learning process—monitoring communication, evaluating her own learning, reflecting on what learning experiences are most effective for her, and more. She is self-aware and aware of the thought patterns of her classmates. She has developed a variety of skills, including operational, procedural, strategic, analysis, and reasoning skills, and she knows when and how to use them. She has learned how to learn.

Analysis-based problem solving, as we have presented here, suggests an approach that removes the tendency to learn without understanding—a tendency that often leads to failure in science. It also brings back an enthusiasm for learning to everyone in the classroom.

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