

## Introducing Rotational Kinematics

### Purpose and Expected Outcome

This is the second in a series of activities that studies rotational motion in analogy with linear motion. The structure of the activities is different from what you are used to. Within the next six activities, each pair of activities will focus on a particular subtopic of rotational motion, namely *kinematics*, *dynamics*, and *energy*.

In this activity, you will begin studying rotational kinematics. You will learn that you already have done some rotational kinematics, though not as formally as you will here. You will learn that rotational kinematics is *analogous* to linear kinematics, in that rotational and linear kinematics share many features, but they are still different. You will learn more about *angular* quantities, such as *angular acceleration*, *angular velocity*, and *angular position*.

### Prior Experience / Knowledge Needed

You should know linear kinematics. You should know how to sketch graphs of position, velocity, and acceleration vs. time, and you should know the relationships among position, velocity, and acceleration. You should know how to write velocity and position as functions of time for constant acceleration. You should be able to use given information about the position and/or velocity of an object to predict its position and/or velocity at other times.

You should know that there are  $360^\circ$  or  $2\pi$  radians in a circle. You should have some experience with angular quantities, such as angular velocity and angular acceleration.

### Explanation of Activity

There are two parts in this activity.

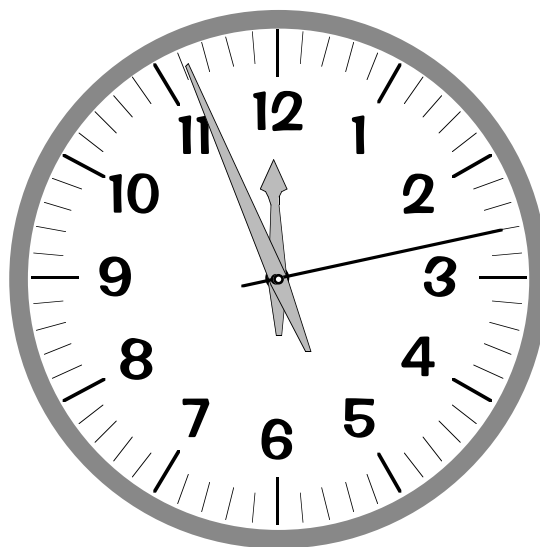
## PART A: Exploring Rotational Motion

Consider a standard wall clock, with an hour hand, a minute hand, and a second hand, as shown below. All questions in this part refer to this situation. You may assume that the clock is about 12in (30cm) wide.

**A1.** Through how many degrees does the second hand move every second? Explain.

**A2.** Through how many degrees does the minute hand move every second? Explain.

**A3.** (a) Through how many radians does the second hand move every second?  
(b) What is the *angular speed* of the second hand in radians/second?  
(c) How would you describe the direction of the movement of the second hand? Explain.



**A4.** A bug is crawling very slowly along the second hand.

- (a) Where would the speed of the bug be the largest? Explain.
- (b) How fast is the bug moving when it is moving its fastest? Explain.
- (c) Does the bug experience an *angular acceleration*? Explain.
- (d) Is it possible for the bug to remain at rest while clinging to the second hand? Explain.

**A5.** (a) Sketch a graph of the angle that the second hand makes with the vertical vs. time.  
(b) At what time(s) is the second hand vertical? Explain.  
(c) What is the slope of this graph at  $t = 17\text{s}$ ?  
(d) Sketch a graph of the *angular velocity* of the second hand vs. time.

**A6.** How does the angular speed of the hour hand compare to the angular speed of someone standing on the Earth (as it spins on its axis)? Explain.

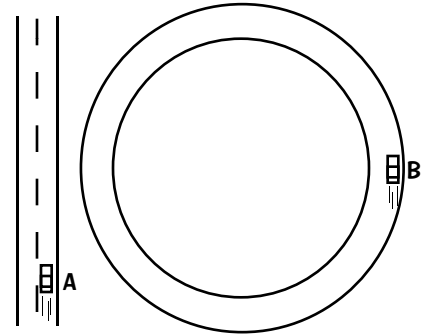
**A7.** How does the angular speed of someone standing on the Earth (as it spins) compare to the angular speed of the Earth traveling around the Sun? Explain.

## PART B: Comparing Linear and Rotational Kinematics

The following situations are paired to give you a sense of how similar angular and linear kinematics are to each other. One involves linear motion, and the other involves rotational motion. Analyze each situation using your own common sense ideas about distance, speed, and time. Do not get bogged down in numerical values.

- B1.** Car A is driving down a long, straight highway at 60mi/h (29m/s). Car B is driving around a circular track, also at 60mi/h. The radius of the track is 50m.

- (a) Which car travels a longer distance in 40 minutes? Explain.
- (b) Which car takes longer to go 80 miles? Explain.
- (c) How long does it take for car B to complete one revolution of the track?
- (d) How many revolutions does car B make in 10 minutes?
- (e) What is the *angular velocity* of car B in revolutions per second? (**Hints:** How many revolutions does the car make in one second? What is the direction of the car's motion?)

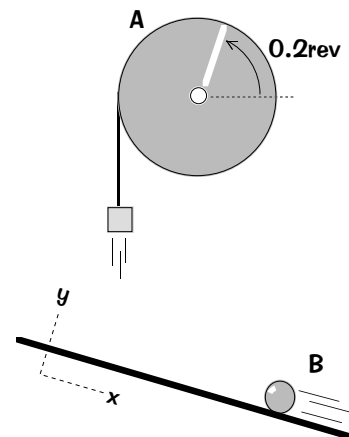


- B2.** Car A is driving down a long, straight highway at 45mi/h (20m/s). Car B is driving around a circular track, also at 45mi/h. (See diagram for B1 above.) The radius of the track is 50m. Both cars accelerate for six seconds until they are going 60mi/h (29m/s).

- (a) Which car has the larger acceleration? Explain.
- (b) What is the change in velocity of car A? What is the acceleration of car A?
- (c) What is the change in angular velocity of car B in rev/s? What is the *angular acceleration* of car B in rev/s<sup>2</sup>? (**Hints:** What is the change in angular velocity every second? What is the direction of this change?)

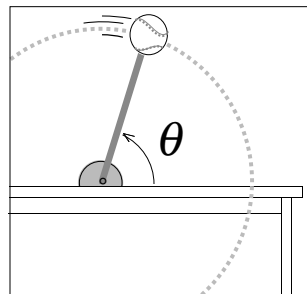
- B3.** At the instant shown, a wheel is spinning at 2rev/s, and slowing down at a constant rate of  $\frac{1}{2}$ rev/s<sup>2</sup>. A white line helps keep track of its position. A ball is rolling up an incline at 8m/s, and slowing down at a rate of 2m/s<sup>2</sup>.

- (a) Which object stops first? Explain.
- (b) Write an expression for the position of the ball as a function of time. (Let  $x_0 = 0.8$ m.)
- (c) Write an expression for the *angular position* of the wheel as a function of time. (Let  $\theta_0 = 0.2$ rev, and assume that the initial angular velocity is negative.)
- (d) Specify the position of each object at  $t = 6$ s.
- (e) Rewrite your expression in (c) using radians.



## Summary

By convention, we measure the angle  $\theta$  in the counterclockwise direction relative the horizontal. So, in the diagram,  $\theta = 72^\circ$ . Counterclockwise (CCW) rotations mean that the angle  $\theta$  is getting more positive, so CCW motion corresponds to a positive angular velocity. In the diagram, the motion is clockwise (CW) so the angular velocity is negative, because the angle  $\theta$  is getting more negative.



## Reflection

- R1.** (a) What is the ratio of the angular speeds of the second hand and the minute hand on a clock? Explain your method for determining this ratio. Compare your method with that of your classmates.
- (b) Which is easier to determine: the ratio of the angular speeds or the individual angular speeds of the two hands? Explain.
- R2.** (a) Estimate the angular velocity of someone in Puerto Rico due to the spinning Earth. Is your answer positive or negative? Why?
- (b) Estimate the angular velocity of someone near the South Pole. Is your answer positive or negative? Why?
- (c) Estimate the angular velocity of the second hand on a clock. Is your answer positive or negative? Why?
- (d) Estimate your angular acceleration (due to a spinning Earth). Explain.
- R3.** Which are harder to answer: questions about linear motion or questions about rotational motion? Why?
- R4.** (a) Is it possible for an object to have a non-zero acceleration and zero angular acceleration? If not, explain why not. If so, give an example.
- (b) Is it possible for an object to have zero acceleration and a non-zero angular acceleration? If not, explain why not. If so, give an example.
- R5.** If something has a negative angular acceleration, is it speeding up or slowing down? Why? Give an example of a situation in which the angular acceleration is negative, but the object is speeding up. Also, give an example of a situation in which the angular acceleration is positive, but the object is slowing down.