Representing Vector Fields Using Field Line Diagrams

Purpose and Expected Outcome

One way of representing vector fields is using arrows to indicate the strength and direction of the field at various chosen points in a region of space. Another way is using *field line diagrams*. Upon finishing this activity, you will know what is meant by a *field line diagram*, you will be able to use it to determine the direction and relative strength of a vector field, and you will be able to reason using it.

## Prior Experience / Knowledge Needed

You should be familiar with electric fields, and you should know how to estimate the electric field at any given location for any given arrangement of charge. You should know that the electric force on a point charge q can be found using $q\mathbf{E}$, where $\mathbf{E}$ is the electric field. You should also know Coulomb's law for determining the electric force.

FIELD LINES AND FIELD LINE DIAGRAMS

A *field line* is a way of representing information about the <u>direction</u> of a vector field. Consider the diagram on the right, which shows 10 field lines in a certain region of space. The direction of the field at a certain point is found using a straight line tangent to the field line at the desired point.
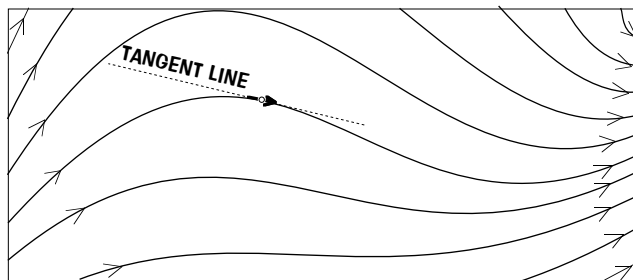
An array of field lines is called a *field line diagram*, and it is a way of representing both the direction and the strength of a vector field. The direction is found by focusing on an individual field line, as above. The strength is found by looking at the density of field lines. For example, the field lines are closest together on the right-hand side, so the
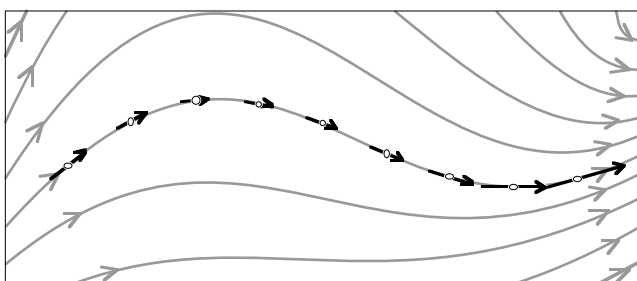
vector field is strongest here. The field lines are farthest apart just to the left of the middle, so the vector field is weakest there. For a point between two field lines, you must estimate the direction and strength of the field.

ELECTRIC FIELD LINE DIAGRAMS

Field line diagrams are especially useful for describing and analyzing electric fields. The simplest charge distribution is a point charge, whose electric field lines are shown to the right. The lines are closest to each other near the charge, so the field is strongest there (as expected). The lines get farther apart as you go away from the charge, so the field gets weaker (also as expected).

Because it is difficult to work with a 3-dimensional field line diagram such as the one on the right, we usually draw instead a 2-dimensional version, as shown below.
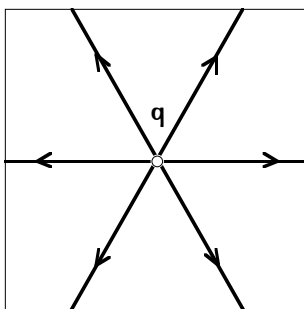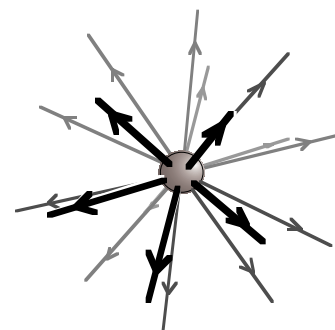
The technique remains the same as above. The direction of the electric field is along the field lines, and the comparative strength is given by the density of lines. (When the density of field lines doubles, the field strength does not necessarily double also, because you are not seeing all the lines. All you can say is that the field gets stronger.)

The number of lines entering or emerging from a point charge is proportional to its charge <u>only in 3 dimensions</u>.

Explanation of Activity

There are three parts in this activity.

PART A: Relating Field Line Diagrams to Other Representations

Consider the following three rows of diagrams. Row A contains 6 arrangements of charge. Row B contains 6 vector field diagrams showing the electric field direction and strength at a variety of locations. Row C contains 6 electric field line diagrams.
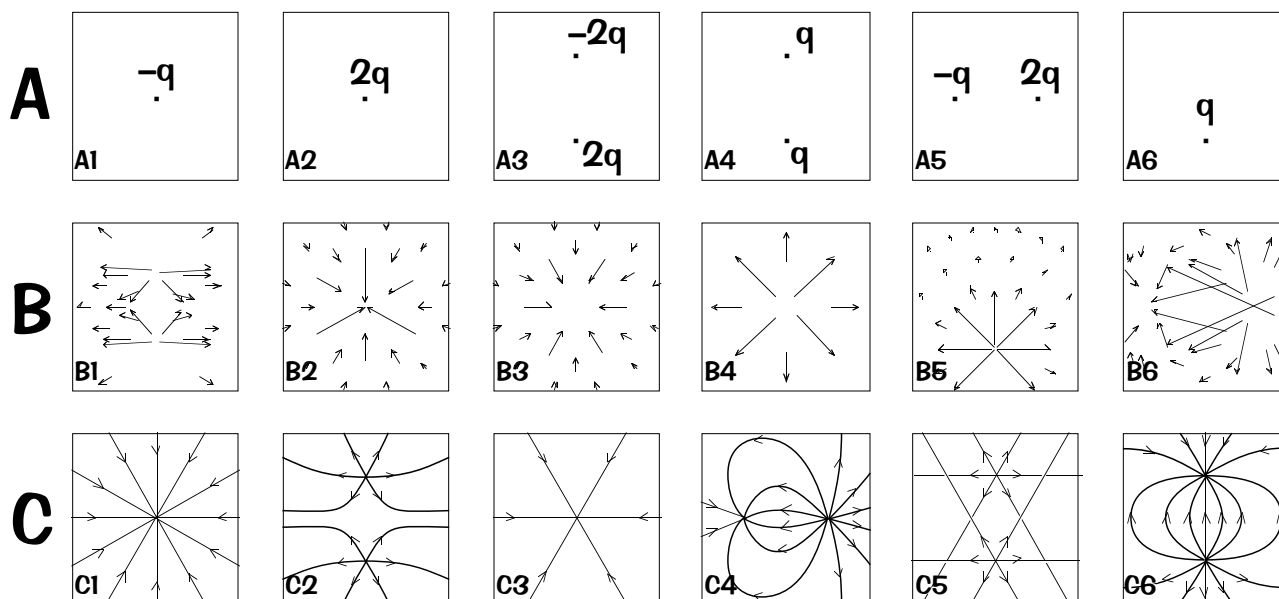

For each arrangement of charge in row A, find <u>at least</u> one diagram from rows B and C that gives its electric field. Be prepared to explain your answers to your classmates.

A1. A solitary point charge $-q$.

A2. A solitary point charge $2q$.

A3. Two point charges, $2q$ and $-2q$, separated by a small distance.

A4. Two point charges, q and q, separated by a small distance.

A5. Two point charges, $2q$ and $-q$, separated by a small distance.

A6. A solitary point charge q.

PART B: Interpreting Field Diagrams

Consider the following electric field diagram for a particular arrangement of charges. Use this situation throughout this part. Be prepared to explain your answers. You may assume that all charges are in the plane of the diagram.
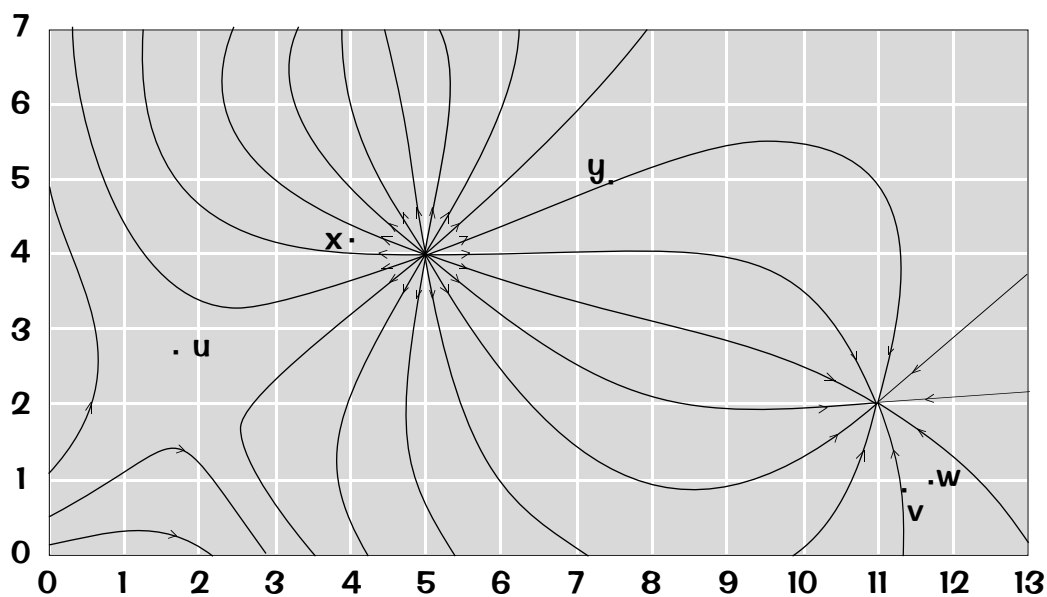
B1. How many charges are located within the 13 × 7 grid shown above? Where are they located? What are the types of charge (+ or –)? Which of the charges in this picture is largest? How do you know?

B2. There is a point charge just outside the picture at the top. What is the evidence? What is the type of charge (+ or –)? What can you say about its magnitude compared to the charges in the picture?

B3. Consider the electric field at the five points u, v, w, x and y indicated in the figure. At which of these points...

   (a) ... is the electric field strongest? ... is the electric field weakest?

   (b) ... does the electric field point closest to the vertical?

   (c) Of points v and w, which has the stronger electric field? Explain.

B4. Why are there no field lines near point u?

B5. If a small test charge were placed 1cm above the grid at some random location, would there be a force on the test charge? Are there field lines in this situation that are not shown in the diagram? Explain.

PART C: Reasoning with Field Line Diagrams

Consider the following electric field line diagram, with 10 locations, each marked with an ✗. Use this situation throughout this part, and be prepared to explain your answers.

Note: You should assume that these charges are held fixed at all times.

C1. How many point charges are there in this situation? What is the type of charge (+ or −) on each one?

C2. On a copy of this diagram, draw the electric field vector at location a.

C3. Would you say that the electric field at location b is stronger than, weaker than, or about the same strength as the electric field at location c?
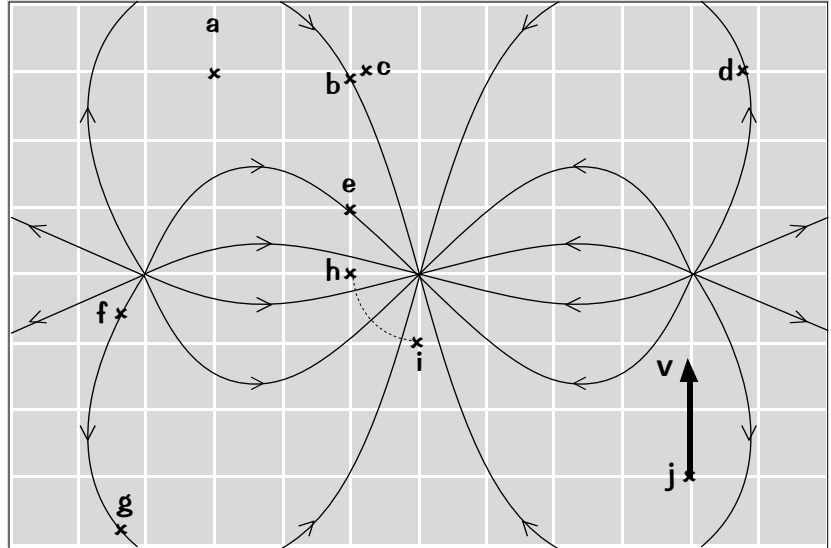
C4. On a copy of the diagram, draw the force on a small negative point charge at location d.

C5. A point charge $-q$ is released from rest at location e. On a copy of this diagram, draw the path of the point charge. (Keep in mind that the other point charges are held fixed.)

C6. A point charge $+q$ is moved slowly from location f to location g along a field line. Is the work done during this process positive, negative, or zero?

C7. A point charge $+q$ is moved slowly from location h to location i along the dashed line shown above. Is the work done during this process positive, negative, or zero?

C8. A point charge $-q$ is at location j with the velocity shown above.
(a) On a copy of the diagram, draw the path of the charge.
(b) When the charge is at location j, is its speed increasing or decreasing? Explain.

C9. Of the 10 locations indicated...
(a) ... at which one is the electric field most horizontal?
(b) ... at which one is the electric field most vertical?

Reflection

R1. Reconsider part A.

   (a) For at least one of the arrangements (in row A), there is no matching vector field diagram in row B. Which one(s)? Is there also a vector field diagram with no matching arrangement? Explain.

   (b) For at least one of the arrangements, there is no matching field line diagram in row C. Which one(s)? Is there also a field line diagram with no matching arrangement? Explain.

   (c) One of the field line diagrams is not valid. Which one? Why do you suppose it is not valid?

R2. If there is no field line at a particular point, does that mean that there is no electric field at that point? Explain why or why not.

R3. Are there field lines other than those shown in a field line diagram? Explain.

R4. Does the field get stronger as you get closer to a field line? Explain.

R5. Reconsider question C4. How would your answer change if the small charge at point d was positive instead of negative? Explain.

R6. If a point charge is released from rest at a location where there are no field lines, what is the motion of the point charge? What if the point charge is released at a location where there is a field line? Is the path of the point charge always along a field line? Explain why or why not.

R7. Do you suppose it is possible for 2 field lines to cross each other? Explain why you do or do not think so.

R8. Which do you think are easier to work with, vector field diagrams (such as those studied in previous activities) or field line diagrams? Explain why you think so.