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ANALOGICAL REASONING VIA IMAGERY: THE ROLE OF TRANSFORMATIONS AND SIMULATIONS

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ABSTRACT

In the classical theory of analogical reasoning, mappings between discrete symbols are a central mechanism in analogy evaluation and transfer. In this study several other analogy evaluation strategies are identified in expert think aloud protocols: bridging analogies, conserving transformations, dual simulations used to detect perceptual-motor similarity, and overlay simulations (Clement, 2004, 2008). These findings add to evidence for the hypothesis that analogical reasoning processes can be imagery based.

INTRODUCTION

Research on scientific reasoning processes of expert subjects has indicated that they can use a variety of methods to generate analogies spontaneously when solving unfamiliar problems (Clement, 1988), and that evaluating the validity of such analogies is essential to using them (Clement, 1989). Even if one has generated a confidently understood analogous case, one must evaluate one's confidence in the aptness or validity of the relation of analogy between the target problem and the analogous case (here called the validity of the 'analogy relation') to have confidence in inferring results in the target. The classical theory of analogical reasoning (Gentner, 1983; Holyoke and Thagard, 1989; Forbus, et al, 1997) focuses on mappings between discrete symbols as a central mechanism of analogy evaluation and transfer. This study identifies several other analogy evaluation strategies observed in expert think aloud protocols, refining and building on Clement (2004, 2008).

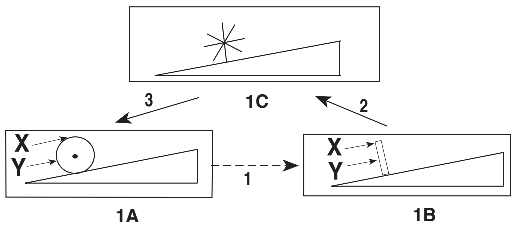

Figure 1. Analogies for Sisyphus problem.

The "Sisyphus problem" illustrated in Figure 1A is an example of a problem where analogy evaluation can be important: "You are given the task of rolling a heavy wheel up a hill. Does it take more, less, or the same amount of force to roll the wheel when you push at X, rather than at Y? Assume that you apply a force *parallel to the slope* at one of the two points shown, and that there are no problems with positioning or gripping the wheel. Assume that the wheel can be rolled without slipping by pushing it at either point."

Motivation comes from observations such as subject S7 gesturing in Figure 3 as he thinks about pulling on the top of the wheel (which he considered equivalent to pushing at X in Figure 1A), and similar gestures were recorded as he made an analogy to pulling on the rope of a pulley. Episodes like these, where the subject gestures as if acting on a real object, convey very strongly a sense that the subject is mentally simulating different cases and challenge us to analyze the role of imagery and simulation in analogy use.

This paper focuses on case studies of two mathematicians working on the Sisyphus problem. The data base they are drawn from comes from a set of interviews with professors and advanced graduate students in scientific fields who were asked to think aloud about a variety of problems (Clement, 2008). The purpose of a generative case study is to develop descriptors for new observations that can motivate the development of new hypothesized process models that have initial grounding in those observations.

CASE STUDIES OF EVALUATION

Subject S2 proposed the analogy that the wheel acts like a heavy pole or lever perpendicular to the slope, with its pivot point or fulcrum at the point of contact with the ground, as shown in Figure 1B. He stated that the lever would be easier to pivot to a vertical position by pushing at X, not Y, suggesting that the same would be true for the wheel. However, in the wheel the point of contact is moving, but the pivot at the bottom of the lever is fixed. This difference calls into question the validity of the lever analogy. In addition some subjects assume that the fulcrum should instead be at the wheel's center. Therefore the evaluation of the validity of the analogy relation (shown as the dotted line between A and B in Figure 1) was in question. This is distinguished from the subject's confidence in predicting the answer for the analogous case B itself, which was quite high in this case.

Evaluation Method 1: Bridging Analogies

One method used by this subject for evaluating this analogy was to generate the bridging analogy, shown in Figure 1C, of a wheel made only of spokes without a rim. Pushing at the top of this wheel should be easier just as in the lever case. And with many spokes, the rimless wheel appears to be the same as the original problem case. By breaking the problem of confirming a "farther" analogy into the problem of confirming two "closer" analogies, such a bridge can make it easier to develop confidence that the wheel does work like the lever in Figure 1B (a correct analysis). Bridging analogies are defined as occurring when the subject finds or generates an intermediate case which shares features with both the target and base. Their value has been documented previously in a number of expert problem contexts and in instructional applications (Clement, 1993; 1998). While it can be very helpful to subjects, bridging in itself is an incomplete strategy for analogy evaluation, since each half of the bridge is a new analogy to be evaluated. Therefore bridging is most useful in conjunction with other evaluation methods. This raises the problem of why experts bother to consider bridging cases at all, since they seem to create more work.

Method 2: Conserving Transformations

In this section I present examples of a second analogy evaluation strategy called conserving transformations and argue that it is different from evaluation methods based on mapping discrete features. A transformation is an action that changes a system 1 to system 2. If 2 is the same as 1 with respect to a feature or relationship R, then the transformation conserves R. A paradigmatic case of three *area conserving* transformations (although he did not identify them as such) is Wertheimer's (1959) method for determining the area of a parallelogram by cutting one end off , moving it to the other end, and rejoining it to form a rectangle.

Example 1. An example of a conserving transformation in the Sisyphus problem occurred when a second subject S7 changed the Sisyphus problem to an analogous one involving an almost-vertical cliff with gear teeth, as in Figure 2b:

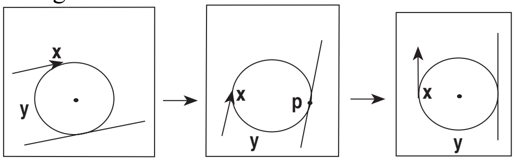

Figure 2: First Wheel analogy series of S7.

(Brackets in transcripts below denote the author's interpretations from viewing tape, while parentheses denote observed actions.)

01 S7: "Suppose it were tilted steeply and you did that; so steep as to be almost vertical. (Draws Figure 2B). It seems like it [the wheel] would skid out from under you the other way [down along the cliff]. This (moves hands as if turning an object clockwise) would get away from you here [at point p]. Let's assume it's gear toothed [gear teeth on the wheel and the cliff] and that it won't slip. "

This subject goes on to imagine a completely vertical case in figure 2C and that analogy does trigger important ideas that are part of his path to a correct solution to the problem, to be discussed later in this paper. I focus here first on the change from situation A to B in Figure 2: it appears to be a double transformation consisting of the change of slope, and the addition of gear teeth. One can define the "targeted relationship" as the one for which an explanation or prediction is sought in the target situation (e.g. the relation between the force required and its location on the wheel). In his further work on the problem S7 never questions the validity of the transformations to an almost vertical slope with gear teeth, and assumes that the targeted relationship in the problem situation is not affected by them—i.e. that they are conserving transformations. (Certainly if the problem were about eating dinner at a sloping table, these transformation would not be conserving.) A traditional approach to analogy evaluation focuses on determining that multiple structural similarities between the base and target are sufficiently *important*. In contrast, a conserving transformation strategy need only focus on determining that a single transformation from base to target is sufficiently *unimportant* (irrelevant to the targeted relationship). Consequently the evaluation of an analogy via a conserving transformation has the potential to require considerably less work than evaluation via mapping.

In the case of adding gear teeth, the transformation is a standardized one used in physics. In the full transcript, it appears that the rotational transformation is intuited by S7 to be irrelevant to the relationship of interest in the problem, i.e. it is a conserving transformation. The origins of this kind of intuition have been studied since Piaget's early conservation experiments but are still poorly understood.

Use of Imagery. The depictive gesture over the drawing, underscored in Line 1 above, provides one source of evidence on the use of dynamic imagery. Although the drawing can be an external support for a static visual representation, it does not depict movements, so it is reasonable to hypothesize that the subject is performing a mental imagistic simulation of the wheel slipping down on the cliff. How might the transformations aid in problem solving? The change in slope simplifies the problem by changing it to one in which forces act mostly along only one dimension: upward and downward. Since the problem already specified no slipping, the gear teeth do not add new information but may help in imagistically simulating what will happen in the analogous cases in Figure 2B or 2C. I call the latter an "imagery enhancement" strategy (Clement, 1994; 2008; in press). (Note: The transformations appear in this case to be a means of not only evaluating the new analogy but also of generating it. In an earlier video-taped think aloud study, Clement (1988) found that of a collection of 31 spontaneous analogies generated by ten experts solving a different unfamiliar problem, a greater number of analogies were generated via transformations than those generated via an association to another case already in memory. However, the present paper focuses on the possible analogy evaluation function of transformations rather than on their analogy generation function.)

Rotating the problem is an example of a continuous transformation; adding teeth is a discrete transformation, as is the following one.

Example 2. As a second example of a conserving transformation, S7 later imagines pulling instead of pushing on the wheel at X in Figure 2a, saying:

S7: "I'm imagining something that's extraordinarily heavy...and I've got my full (holds both hands out as if pulling something and shakes them slightly) power available- and where would I apply that? My instinct tells me [it is easier to apply force at] X again but that er, but again it's in terms of a pull and not a push. I'd have to get a grip. Assuming that's not a problem, then pulling should be the same as pushing."

Although to a physicist, the equivalence of pulling and pushing might be obvious here, the subject is not a physicist and revisits this issue as a question more than once, so it is considered to be an analogous case here. Here again, the subject appears to have evaluated the analogy relation between two situations by viewing their difference as a conserving transformation of a single feature that will have no effect on the targeted relationship in the problem.

Griffith, Nersessian, and A. Goel (2000) have designed and investigated a computer program which successfully accounts for a number of features of other protocols in our data base described in Clement (2008). Transformations played an important role in evaluating, modifying, and improving faulty analogies or models in their program.

Method 3: Dual Simulation

There is also evidence in the protocols for a sometimes imprecise but very direct strategy for analogy relation evaluation termed "dual simulation". Here dual simulation means running two different simulations and comparing them to each other. For example, one might run simulations of the wheel and the spoked wheel without a rim in Figure 1A and 1C to determine that they "roll" approximately in the same way, especially in the case of a large number of spokes. In a dual simulation, simulations of the target and the analogous case are run in as much detail as possible, either sequentially or simultaneously. The dynamic images of the behavior of each system are then compared; if their behavior "appears" to be the same, the aptness of the analogy relation receives some support, depending on the level of certainty in the comparison.

Imagistic Simulation. Dual simulation is assumed to depend upon a double application of the process of *imagistic simulation* described in Clement (1994, 2008, in press). Those studies found evidence for this internal process from several observation categories for external behavior: *personal action projections* (spontaneously describing a system action in terms of a human action, [consistent with the use of kinesthetic imagery]), *depictive gestures* (depicting objects, forces, locations, or movements of entities), and imagery reports. The latter occur when a subject spontaneously uses terms like "imagining," "picturing," or "feeling what it's like to manipulate" a situation. (The term imagery is used in a broad sense here that includes all perceptual modes plus kinesthetic imagery of actions). In several of the present cases one sees *dynamic imagery reports* (imagery reports involving movement or forces). None of these observables are infallible indicators on their own, but each provides some evidence for imagery (denoted by underscoring in transcripts in this study.) Taken together with the subject's new predictions from simulating a case, the observations above can be explained as the product of an *imagistic simulation process* wherein a somewhat general perceptual motor schema assimilates the image of a particular object and produces expectations about its behavior in a subsequent dynamic image, or simulation.

Pulley Case. A second example of a dual simulation occurred as subject S7 made an analogy to a pulley using the diagram in Figure 2C. Ordinarily a pulley wheel is attached to a separate load, but here the pulley wheel is large and heavy enough to act as its own load. He appears to make an initial evaluation of whether these two systems are similar in the following passage:

05 S7: "What it feels like is the weight of it [wheel in Figure 2B]-; is pretty close to parallel with what you've got if you go roll it with a complete vertical. It now begins to feel like a pulley, is what it feels like..(Draws Figure 2C) What the vertical [ramp or wall on the right in 2C] is over here no longer matters perhaps but we'll say it's er, gear toothed again.

06 S: ...And you're over here pulling like this [on the wheel at x in Figure 2C]. That feels like you're on the outside of a pulley pulling up. "

Here the subject appears to run simulations of both the wheel on a vertical ramp system and a pulley system and to make a global comparison of the behaviors of the two systems. The personal action projection and the kinesthetic imagery report in line 6 provide evidence that imagistic simulations are occurring, and these give some support to the hypothesis that a dual simulation is occurring to compare the two cases. Again, the drawing may provide support, but cannot provide perceptions of force or movement. One can hypothesize that the subject is running a simulation by projecting an image of pulling and movement into diagram 2C for each case and comparing at least the gross behavior in the two simulations. In this view one thinks of rolling a wheel or using a pulley as perceptual motor actions. These may be controlled by perceptual motor schemas that include motor control processes that are in parallel control of many muscles, as opposed to being discrete symbol structures. Part of the question being considered is whether analogies can occur at that presymbolic level of representation. The basic idea behind dual simulation is that just as one can imagine whether one can throw an orange peel farther than a potato chip , one can compare imagined events and say whether they are about the same.

It could be argued that the wheel on the vertical slope simply accesses the pulley idea in memory, with no need to hypothesize a process of 'dual simulation'. However, the subject's statement, "Now it begins to feel like a pulley" and "feels like you're on the outside of a pulley pulling up" suggest that more than activation is going on and that he is thinking about similarities between simulations of the two situations.

A dual simulation may establish the analogous case as being relevant and plausibly analogous in that its behavior is similar, at least at a gross level of qualitative behavior, to the target. But this does not yet tell the subject confidently whether the two systems exhibit the same targeted relationship between point of application and amount of force needed. Thus in the above cases dual simulation appeared to serve only as a check on the initial plausibility of the analogy. One then needs to be clear that dual simulation as an analogy evaluation strategy does not necessarily mean confidently simulating the *targeted relationship* independently in both base and target. In that case there would be no need for an analogy because the target could have been directly simulated on its own. However, it is plausible that dual simulation can still help one determine whether the target and base are similar with respect to other important behaviors, thereby increasing one's confidence that the analogy is sound (or eliminating the analogy from consideration).

Overlay Simulation Process

Lever case: One can also point to evidence for the existence of a more precise type of dual simulation that I term "overlay simulation" where the image of one simulation can take place "on top of" a second image. An overlay simulation is said to occur when two systems are simulated at appropriate relative sizes so that spatial features within them can be spatially aligned. This may involve imaging them simultaneously as overlapped or combined or in frequent alteration at the same position within one's 'internal visual field". Although I have separated them in Figure 1 for clarity, S2 actually drew his lever analogy (Figure 1B) directly on top of the wheel (Figure 1A) and compared the movement of the wheel and the lever. This meant that the arrow symbolizing the application of a force by pointing to the top of the wheel was also pointing to the top of the lever. When two separate systems are represented as overlapping in the same external diagram with salient features aligned I term this an *overlay diagram*. This supports the interpretation that internal dynamic images of the two systems and their actions were overlapping in the same way. Presumably the alignment of key features made it easier for him to compare the expected movements and resistances of the wheel and the lever as he simulated each of them.

***Spokes case**:* Overlay simulation may also be responsible for the power of S2's "spoked wheel without a rim" bridging analogy shown in Figure 1C. For the spoke that is touching the ground, the spoke can be seen as a lever with its fulcrum at the ground. This means that the entire wheel of spokes can be seen at any one time as equivalent to a single lever, supporting the analogy on the right hand side of the bridge BC in Figure 1. This subject spoke of a tireless, rimless wheel. Again this is shown separately in figure 1C for clarity, but in fact the spokes were inscribed to fit on top of and aligned with a circular wheel in the subject's drawing. So on the other side of the bridge, AC in Figure 1, the spokes are envisioned at the same size as the original wheel, and this may make it easy to sense via dual simulation that they behave in the same way as the wheel when a force is applied. In particular, the way the rimless spoked wheel rolls can be seen as similar to the way the original wheel rolls. That is, it appears, especially with many spokes, to have the same kind of motion in a dual imagistic simulation and therefore be amenable to the same type of analysis with respect to the causes of motion. Although such arguments can be bolstered mathematically to make them rigorous, as a form of heuristic reasoning, this type of qualitative perceptual motor argument can sometimes be quite compelling.

***Pulley case**:* To continue with S7's pulley protocol, it is also true that the references to the wheel on a vertical ramp case and the pulley cases are actually made in reference to only the single drawing in Figure 2C. As part of an attempt to evaluate that analogy, S7 continues to speak and gesture as if alternating between seeing the same drawing (Figure 2C) as a wheel and a pulley, as noted in parentheses in the transcript below. Continuing from line 06 above:

> 06 S7: (Referring to wheel in Figure 2c): "And <u>you're over here pulling</u> like this [at X].
> (Referring to pulley):That <u>feels like you're on the outside of a pulley pulling up</u>. "
> 07 S: (Referring to wheel):And since you say it doesn't slip, then this thing over here (points to line in upper right of Figure 2C and adds upward pointing arrowhead to it) must be providing the other half of it, <u>something it feels</u>. (Referring to pulley):In which case it's a classic pulley; (Referring to wheel):no, it can't be classic pulley. (Referring to pulley):But it's, like a classic pulley in which now you only need half of the force. (Referring to wheel):if the weight of the thing is 10 lbs. here, it feels like 5 would work here (writes 5 on upper left of C) and 5 over here (writes 5 on upper right) (Referring to pulley):as though it were a pulley… So let's imagine it is a pulley.
> 08 S: (Referring to wheel):[In] this new point of view, <u>it feels like working at X</u> [on the edge of the wheel] <u>is better</u> [than at the center]."

The underlined kinesthetic imagery reports and alternating references to both the wheel and the pulley systems while staring at the same diagram 2C provide initial evidence for an overlay simulation here that compares the system of rolling the wheel straight up a vertical cliff to a pulley system. Presumably it is easier in an overlay simulation to switch rapidly between simulations of the two cases, as in the passage above. Evidence for kinesthetic imagery is indicated by phrases like "feels like you're on the outside of a pulley pulling up" and "you're over here pulling" in the transcript, and such imagery is clearly not already enacted in the static drawings.

Later he adapts the pulley analogy by laying it on its side diagonally on the ramp in Figure 4B, [with the rope running over the top of the pulley wheel], but he still expresses some reservations about it: “This rope wrapping around here...doesn’t feel to me necessarily like...pushing (moves hand l. to r.) on the outside of a wheel.” This skepticism is certainly understandable given that the rope exerts a force at many points along the back of the pulley wheel, whereas the hand only exerts a force at one point. However, in the passage below he appears to reevaluate the analogy positively by (1) generating a bridging analogy; and (2) using dual simulations. Therefore this final example is more complicated because it combines these two strategies.

Figure 3. (S7) “It’d be a lot easier to hold it here.”

S7 first generates a new bridging analogy of a rope attached to a gear-toothed wheel at X in Figure 4C:

> 162 “Seems clear that- (holds both hands out as if pulling a rope for 4 sec.) hang on up there! So we attach a rope to one of the teeth [as in 4C], now it becomes more like the pulley problem [in 4B] (holds r. hand out as if pulling a rope for 3 sec)…the teeth at the bottom [in 4C] are playing the role of-; the pulley doesn't look so bad after all. And you hang on for all you’re worth up there, to keep it from rolling; to keep it balanced. “

Here he first focuses on confirming the analogy between cases 4B and 4C on the right side of the bridge in Figure 4. Evidence for dual simulation of these two cases comes from the juxtaposition of: (1) depictive gestures as evidence for imagistic simulations; (2) his attention alternates rapidly between the cases. He continues by evaluating the analogy between A and C on the left hand side of the bridge:

> 163 S7: Seems a lot easier than getting down here behind it [at "Y" in Figure 4A] and pushing. Why? because of that coupling pulley effect. It seems like it would be a lot easier to hold it here [near X in 4C] for a few minutes (Holds hands in “pulling” position in Figure 3) than it would be to get behind it… yeah, my confidence here is much higher now, that it's right… [easier to push at X in 4A]
>
> 164 S7: And so the pull--it just felt right with the pulley feeling. Now pushing (lays extended finger on paper pointing up slope at X in Figure 4A and moves it toward X) uh,.. it’s got to be the same problem…
>
> 178 I: Do you have a sense of where your increased confidence is coming from?
>
> 179 S7: It’s the pulley analogy starting to feel right.

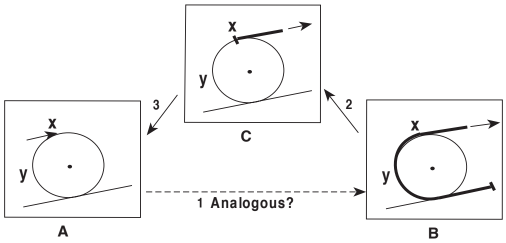

Figure 4. (A) Push or pull on wheel; (B) Pulley rope around wheel (C) Rope attached to wheel with gear teeth;

S7 appears to have gradually transferred perceptual motor intuitions about pulleys to the original problem. In line 163 the subject appears to focus on whether a force applied to the wheel at Y in 4A and a pulling force applied at X in 4C “feel” the same as he performs an imagistic simulation of each case twice in alternating fashion. This provides evidence for another dual simulation, as does the imagery report in line 164, “it just felt right with the pulley feeling.” Furthermore, although I have drawn three cases in Figure 4 for clarity and to illustrate the bridging strategy, in fact S7 stared only at Figure 4A while talking about the three cases; he did not actually draw 4C and 4B, but apparently projected images of the attached rope and pulley cases into the diagram. This suggests that the dual simulations here were actually also overlay simulations, supported by a single diagram. One can hypothesize that internal overlay simulations create a context whereby the imagistic alignment of forces or movements in imagistic simulations of different cases, as well as the evaluation of the validity of the analogies between the cases, can be more easily made.

The bridging case in Figure 4C of a rope tied to the wheel at point X appears to serve the purpose of setting up two pairs of cases (A:C and C:B) that are more perceptually similar than A:B. This may be an important advantage if it makes it possible for the evaluation of each pair of cases to be done via a conserving transformation or a dual simulation. This provides one answer to the earlier question of why bridging can be useful to a subject even though it seems to add more work in creating additional analogy relations. In addition, overlay diagrams appear to support overlay simulations that also make evaluation via a conserving transformation or dual simulation easier.

DISCUSSION

In summary, rather than a single process for mapping elements in a discrete symbolic representation, a number of additional processes for evaluating an analogy relation have been identified, namely: bridging analogies, conserving transformations, dual simulations to detect dynamic similarity, and overlay simulations. Roughly, dual simulations work by allowing the subject to detect a perceptual motor similarity between base and target. Overlay simulations are a special type of dual simulation in which the image of one case is aligned over the other case to make comparisons more precise. Conserving transformations work by allowing the subject to detect the causal, perceptual motor irrelevance to a targeted relationship, of making a transformation that changes the target to the base or vice versa. An intermediate bridging case is a higher order strategy that can facilitate making one of the above processes easier to perform.

The hypothesized way in which these processes could be coordinated is summarized in the outline of an idealized prescriptive algorithm for evaluating an analogy, shown in Table 1. The algorithm defines three procedures which can call each other. "Fail" here means "failure to confirm the analogy relation". The algorithm shows bridging as a higher level strategy, whereas the other evaluation methods are more basic and direct ways of determining similarity and analogical validity; i.e. bridging is seen as a strategy for helping the other methods work. The algorithm is also recursive, to allow for multiple bridges.

- •Evaluate Analogy Relation (T, A)
  - •Direct Evaluation (T, A)
  - •If fail, Bridge (T, A)
  - If fail, quit

- •Direct Evaluation (T A)
  - •Dual Simulation of (T, A) to evaluate perceptual motor similarity (may only provide initial plausibility)
    - -May be enhanced by Overlay Simulation
  - • and/or Find Conserving Transform from A to T
  - • and/or Evaluate Mapping of (T, A) (e.g. as in Forbus, et al, 1997)

- •Bridge (T, A)
  - •Generate bridging case B
  - •Evaluate (T, B); if fail, try a different bridging case B above, or quit
  - •Evaluate (B, A); if fail, try a different bridging case B above, or quit

Key:
T = Target Case; A = Analogous (Base) Case
B = Bridging Case

Table 1. Strategy Using Multiple Methods For Evaluating An Analogy Relation

Such an algorithm would need to track an accumulating confidence score for the validity of an analogy relation, reflecting contributions from each method, until the score was higher than a given confirmation threshold (or not).

Mapping

Why should one consider not restricting the model of analogy evaluation to a process based on symbolic mapping? To me, that limitation would make it difficult to explain: (1) the numerous depictive gestures and kinesthetic imagery reports; (2) S7's repeated kinesthetic references to whether the wheel "feels like" a pulley; (3) the subject appearing to use the latter as his main criterion for whether he accepts the pulley analogy; (4) the need to break the original analogy down into several pieces using multiple bridging analogies; (5) the use of overlay diagrams for analogies that appear to support the precise visual alignment of features such as the movement of a lever projected onto the wheel. It is possible that the imagistic analogy evaluation mechanisms identified could operate prior to mapping via discrete symbols, although the relationship between these processes and mapping is still unclear. But when subjects can articulate such mappings, that may add another important kind of precision to the process of analogy evaluation. Subjects who can use both kinds of processes to support each other may have the greatest advantage.

Scientific Models

I refer to *analogical projection* as a form of analogical transfer where not just discrete symbolic features, but imagery and simulative elements are transferred from one case to another. As opposed to the evaluation processes emphasized in this paper, projection refers to a desirable end state wherein, for example, a subject may come to see the wheel as a lever, or series of levers, as their final qualitative model of the wheel. The final form of a successful projective analogy is then hypothesized to be a precise form of overlay simulation of aspects of the base projected onto or into a simulation of the target's behavior. Dynamic imagistic spatial alignment can be critical in projection; e.g. in seeing the wheel as a lever, it is critical for the lever to rotate around the point of contact with the ramp rather than (incorrectly) around the center of the wheel. Therefore it makes sense that a successful projective analogy would most likely use an internal form of overlay simulation where, for example, the motion of the lever can be simulated 'on top of' the motion of the wheel for precise imagistic alignment. This is suggestive of a mechanism for explanatory models in science, such as the elastic particle model of gases; wherein an image of colliding particles may be projected into the image of a heated gas driving a piston. Such overlays may be central for qualitative modeling in science. Clement (2003, 2008) extended this theme by examining evidence for the transfer of imagery and runnability from source analogues to explanatory models and hypothesized that this may be an important source of model flexibility, providing an argument for the importance of such processes.

Implications

The importance of bridging analogies as an instructional technique has been documented previously (Clement, 1993; 2008), and the same may very well be true for conserving transformations (Wertheimer, 1959), and overlay diagrams/simulations.

Findings in this paper add to previous evidence (Casakin and Goldschmidt, 1999; Clement, 1994, 2003, 2004, 2008: Craig, Nersessian and Catrambone, 2002; Croft and Thagard, 2002; Trickett and Trafton, 2002) for formulating the general hypothesis that many analogical reasoning processes can be imagery based.

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