Computing the Potential Energy

Purpose and Expected Outcome

You will learn how to compute the potential energy for gravitation and for ideal springs.

Prior Experience / Knowledge Needed

You should know the definitions of work and kinetic energy, and you should know the Work–Kinetic Energy Theorem. You should be familiar with the concept of potential energy. You should have some ideas about what factors affect the gravitational and the spring potential energies.

GRAVITATIONAL POTENTIAL ENERGY

With an object near the surface of the earth, the change in potential energy is the work done against the earth's gravitational force to change the height of the object. Mathematically, we write:

$$\Delta U_g = F_g \Delta y = mg\Delta y \qquad \text{(near the surface of the earth)}$$

where m = mass of the object

g = gravitational constant (about 10N/kg near the earth)

Δy = change in height of the object

To find the potential energy, we must first define a *reference height*. This is not always the origin. It is the height at which U_g is assigned the value of zero. Any reference height may be chosen, but you must use the same one for the entire problem. You must decide what is the best choice for any particular situation. Often, we set U_g to be zero at height $y = 0$. With this choice for the reference height, the gravitational potential energy can be written:

$$U_g = mgy \qquad \text{(near the surface of the earth relative to chosen reference)}$$

ELASTIC POTENTIAL ENERGY (SPRINGS)

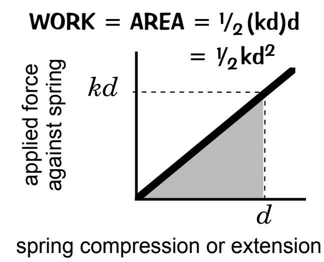
Even though an ideal spring does not exert a constant force, it does exert a predictable force, depending on its spring constant and how far it is compressed or stretched from its relaxed condition. Unlike gravitation, springs have a well defined reference length: its relaxed length. When relaxed, the spring exerts no force, it can do no positive work, and it cannot increase the kinetic energy of anything. Therefore, when the spring is relaxed, we say the spring has zero potential energy.

By pulling or pushing on a relaxed spring, we do positive work on the spring, increasing its potential energy. The potential energy of an ideal spring is:

$$U_s = \frac{1}{2}kd^2 \qquad \text{(for an ideal spring having spring constant } k\text{)}$$

where d = distance the spring is stretched or compressed from its relaxed state.

(The factor of ½ is very important. It can be seen in the diagram to the right, because the work done on the spring is the area of the gray triangle.)



Sometimes it is more convenient to write the elastic potential energy in terms of the spring's actual length:

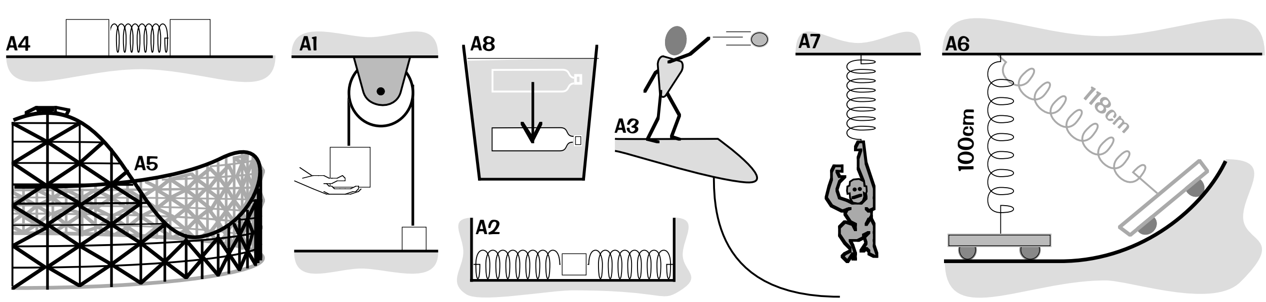
$$U_s = \frac{1}{2}k(L - L_0)^2 \qquad \text{(for an ideal spring having spring constant } k\text{)}$$

where L = length of the spring
L_0 = unstretched or "relaxed" length of the spring

In general, <u>you</u> must decide which form of U_s is best for the situation you are studying.

Explanation of Activity

In this activity, you will compute values for the potential energy and the change in potential energy for gravitation and for ideal springs in different situations. Diagrams of all the situations are shown below.



A1. A 2kg block and a 1kg block are attached to opposite ends of a string, and everything is placed on a pulley as shown. Initially, the 1kg block is on the ground and the 2kg block is 0.4m above the ground. After the 2kg block is released what is the change in potential energy of the system made up of the two blocks, the string, the pulley, and the earth.

A2. A 200g block is attached to two identical springs as shown. Initially, the springs are unstretched and are 20cm long. Each has a spring constant of 10N/cm. If the block is moved 10cm to the right, what is the total potential energy in the system.

A3. An 80kg person throws a 0.4kg ball off a small hill. The ball is released 1.5m above the top of the hill, and the hill is 2m high. Using the top of the hill as the reference height, what are...

(a) ... the potential energy of the earth–ball system immediately after it is released, and

(b) ... the potential energy of the earth–ball system when it hits the ground?

A4. A 30cm spring having k = 6N/cm is compressed to half its original length using two 1kg blocks as shown. What is the potential energy of the spring?

A5. A roller coaster car has a mass of 1000kg. During a test run the car travels empty from the topmost point to the end. The total distance traveled is 500m, and the total change in height is 75m. What is the change in potential energy of the system during this run?

A6. A 90cm spring with a spring constant of 0.1N/cm is securely attached to the ceiling. Its other end is attached to a 0.5kg cart, which is placed on a curved incline as shown. (The length of the spring is 118cm at this time.) The cart is released and eventually comes to rest as shown. (Its length is now 100cm.) What is the change in potential energy stored in the spring?

A7. A 40kg monkey hangs at rest from a spring having a spring constant of 25N/cm.

(a) What is the potential energy in the spring?

(b) Relative to the position of the monkey when the spring is unstretched, what is the gravitational potential energy of the earth–monkey system?

A8. An empty 1-liter bottle has a mass of about 80g. The sealed bottle is pushed into a bucket of water so that it is completely immersed as shown by the white outline in the diagram on the left. If the bottle is then pushed down 50cm farther into the water, what are the changes in the gravitational potential energy of...

(a) ... the empty bottle, and

(b) ... the water in the bucket?

Reflection

R1. Can the gravitational potential energy of a system be negative? If so, give an example. If not, explain why not.

R2. Can the potential energy of a spring be negative? If so, give an example. If not, explain why not.

R3. The gravitational potential energy of an object is negative, and it is released from rest. Is it possible for the kinetic energy of the object to increase? Give an example and show how this might happen. What happens to the gravitational potential energy?